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AUTHOR Koch, Dale R.
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ABSTRACT

This book is divided into two parts: a discussion of the role of individualized instruction in today's schools, and a detailed description of objectives and implementation of the Duluth Individualized Mathematics Program. In the first part, the author argues that schools should not be preservers of tradition but agents of change; diversity should be encouraged, both because society needs varied talents and because individuals have varied needs. The role which mathematics can play in developing the individual personality is then discussed. The second part of this book considers first the needs for behavioral objectives in teaching, illustrated by the Duluth Elementary Mathematics Content Guide and over 250 objectives and sample evaluation items. Some practical aspects are then discussed: student learning guides (student contracts), student achievement records, coding procedures, student planning schedules, student record forms, classroom design, and problems of evaluation of student progress. The book concludes with a list of commercial materials of use in an individualized mathematics program, brief descriptions of relevant developmental projects, and an extensive bibliography. (MM)

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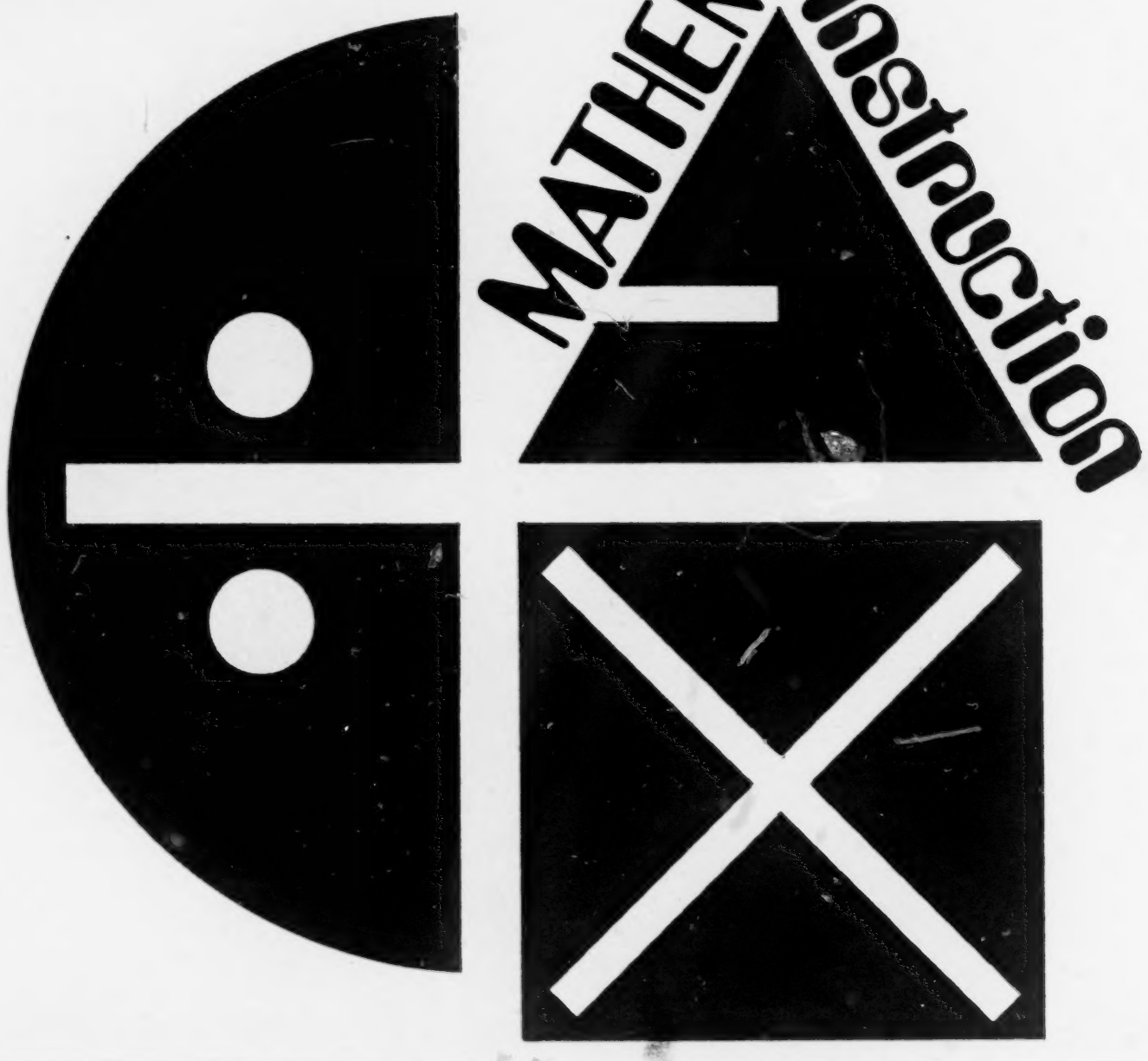

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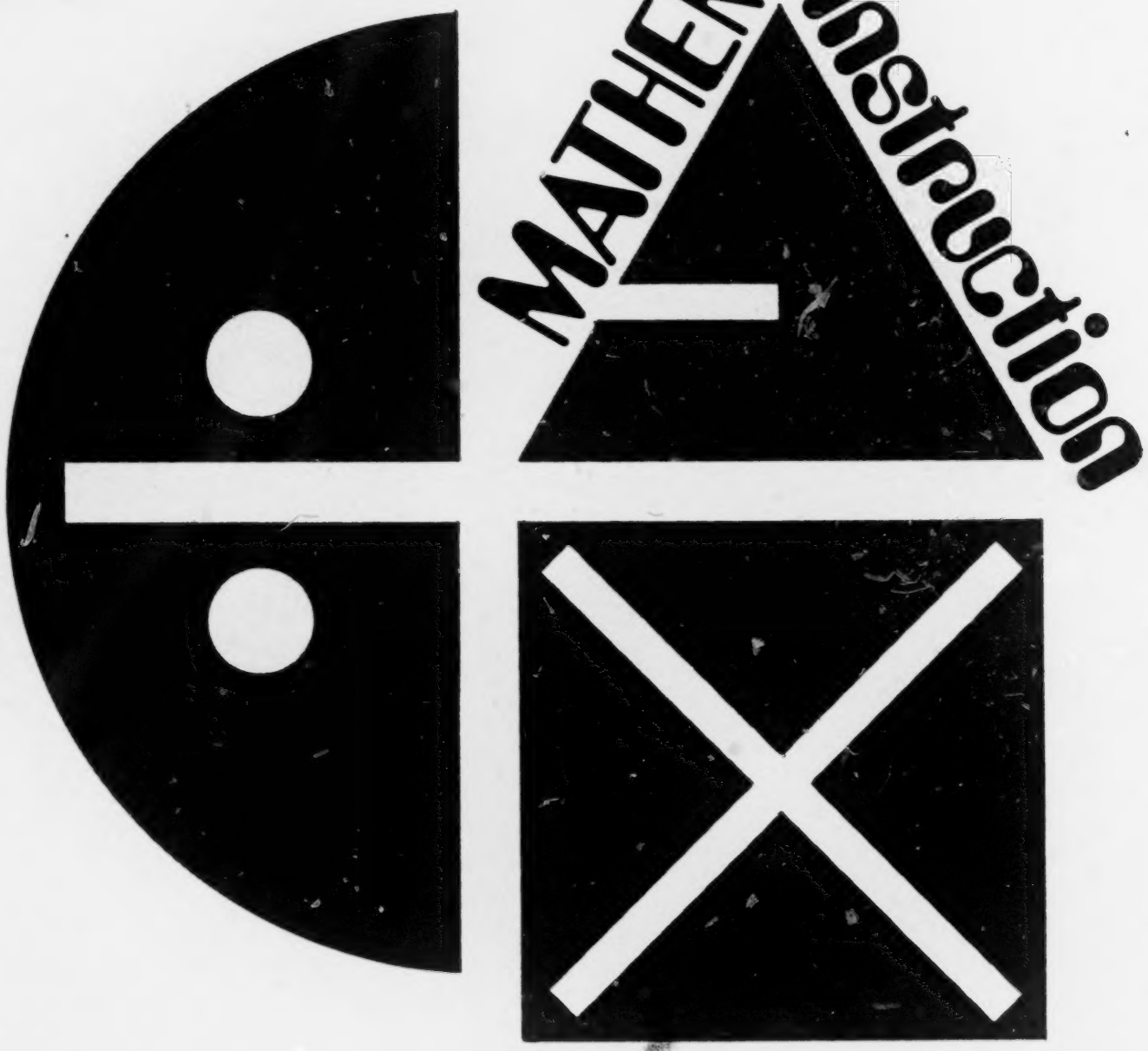

Theory and Practice

BY DALE R. KOCH

SE 013 161

INDIVIDUALIZED MATHEMATICS INSTRUCTION:

THEORY AND PRACTICE

Dale R. Koch

Auburn, Alabama

June 10, 1970

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PREFACE

Ours is a time of criticism. Nearly everyone is critical of the war in Vietnam. Blacks are unhappy with whites, and whites with blacks. The "silent majority" is hardening its attitude toward those who would dissent. Students are driving university administrators up the proverbial "wall". The hungry are not fed. Inflation continues on the upswing. Most recently, almost everyone has gotten on the bandwagon against pollution of the environment and over-population. Neither has education escaped the sharp edge of critical analysis of its many obsolete practices. Noyes and McAndrew state: "As presently organized, the inescapable truth is that our schools seldom promote and frequently deny the objectives we, as a nation, espouse. Rather than being assisted and encouraged to develop their own individuality, our children are locked into a regimented system that attempts to stamp them all into the same mold. The student is filled with facts and figures which only accidentally and infrequently have anything whatsoever to do with the problems and conflicts of modern life or his own inner concern." Criticism in itself is not bad as long as the one who criticizes does not stand in the path of change. Whatever

changes must take place within the educational establishment, and they are many, programs of individualized instruction offer a rational basis for resolution of the problems that have caused the crises in our society.

Ultimately, any program of individualized instruction must encourage and support activities on the part of the student that help him answer the questions:

1) Who am I? (self-identity)

2) What am I doing? (self-orientation)

3) Where am I going? (self-direction)

This process, to be sure, is a progressive one. But unless educators attempt to assist students in answering these questions, a meaningful portion of any individualized program is totally lacking. They will have totally failed the student and his society.

The intent of this book is twofold: to serve public school systems in the establishment of programs of individualized mathematics instruction, and as a resource for the college elementary mathematics methods course.

My thanks to Professor Thorwald Esbensen of Florida State University for his unwavering confidence and support during the developmental stages of individualized instruction in Duluth, Minnesota; to June Brieske, John Downs, and Robert Shaul, for the many significant discussions that helped my educational development; to my wife, Mary Elizabeth, for her understanding during the work in the Duluth program and during the writing of this book; to R. Edwin Wilgus for his design and execution of the classroom environments used in the chapter on facilities; to Bob Capps for his design of the cover; and to Kaylyn Johnson, who along with my wife, proofread the manuscript.

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Noyes, K. J., and McAndrew, G. L., "Is This What Schools are For?," *Saturday Review*, December 21, 1968, p. 65.

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MATHEMATICS INSTRUCTION-WHERE ARE WE?

It is a virtual certainty that if one picks up an education journal with a publication date of 1970, one would find a rather significant portion of its content dealing with the crises existing in the U.S. today - the war in Vietnam, the needs and demands of minority groups including university students, the problems of urban living, the lack of relevancy in our educational programs, and so on. These crises are bringing about changes in our people of which they are, in many cases, unaware. Marshall McLuhan has observed: "The medium, or process, of our time--electric technology--is reshaping and restructuring patterns of social interdependence and every aspect of our personal life. It is forcing us to reconsider and re-evaluate practically every thought, every action, and every institution formerly taken for granted. Everything is changing---you, your family, your neighborhood, your education, your job, your government, your relation to 'the others'. And they're changing dramatically."

The cries of various groups, educators included, for relevant changes in curricula are becoming more vociferous, more ardent and more demanding than ever before. But are significant changes forthcoming? Regretfully, no! It

is true that we have such innovations as programmed instruction, computer-assisted instruction, micro-teaching, and even the most hallowed of all currently popular educational innovations - individualized instruction. But for the most part, these simply dress up old content in a new technology. Witness Pat Suppes' CAI! Why have educators, teachers and administrators, permitted themselves to be blinded by modern technology in the classroom? Why have they failed to demand content changes rather than simple "mode of presentation" changes?

The answer to this question is not easily supplied. However, a large part of the answer centers on the Protestant Ethic that has gripped our educational establishment for so many years. Today's teachers - children during the 30's, 40's, and 50's - grew up under this Ethic and its concept of sin. The attendant subconcepts of right and wrong leads one to view life as black and white, good and bad, no gray permitted. One method is better than another, one textbook better than others, one teacher better than another, and one child better than others. This philosophy in turn leads to a search for the panacea to problems in education. Thus, when modern technology happened on the scene, educators lapped it up. Here was the antidote. All problems would be resolved!! The computer would take care of educational problems now. All would be well!

Naturally, teachers tended to forget about content and viewed through rose-colored glasses and general misunderstanding, all of the wonderful results that would be achieved with students and learning. But, lo and behold, it hasn't worked that way! Utopia has not been reached (achieved). Why wasn't modern technology the key educators sought? The failure rests not with technology but with the philosophy which promoted it as *the* answer.

The resolution of the problem lies rather in the following interrelated ideas:

1. We live in a changing society.
2. The content of curriculum is determined by the culture of a people, the sum total of the pattern of living in a society.
3. Curriculum is constantly changing.
4. To change is to make a material difference so that the thing is distinctly other than it was.
5. Educational change demands application of the decision-making process.
6. The decision-making process requires genuine alternatives from which to choose.
7. An alternative is the option of one or more out of two or more things.
8. Students are different in most aspects of their existence including perception.

9. One method or approach or content will not meet the needs of all students.

Based on these concepts, one must conclude that curriculum development and improvement are based on sets of alternatives from which to choose and that, generally, more than one alternative should be chosen. The fallacy in the "panacea approach" to curriculum change lies in the statement: "One method, approach, or content can be all things to all students." It is high time education rids itself of the immature notions of rightness and wrongness of a method, of a text, of a teaching approach, of appropriate or inappropriate content. Educators must seriously consider the generation of significant alternatives to content, method, and materials, if we are to achieve individualized instruction for each child. Some progress in providing alternatives is being made.

In the March, 1970 issue of the *Phi Delta Kappan*, an article appeared by Donald W. Robinson entitled "Alternative Schools': Challenge to Traditional Education?" In the article reference is made to the New Schools Exchange which describes itself as the only central resource and clearinghouse for all people involved in "alternatives in education". This exchange provides a sense of community among radical educators. These educators feel regimented by an unsympathetic administration in a sterile learning climate. The Exchange

lists several hundred innovative schools that supposedly are offering alternatives to traditional education.

Alternatives in education are also developing outside the U.S., noteably in England. Beatrice and Ronald Gross, writing in the May 16, 1970 issue of the *Saturday Review*, discuss the British Infant Open Classroom. The Open Classroom "refers to a new approach to teaching that discards the familiar elementary classroom setup and the traditional stylized roles of teacher and pupil, for a far freer, highly individualized, child-centered learning experience that may hold the key to a radical reformation of primary education." Focus of the Open Classroom is on "a general atmosphere of excitment; virtually complete flexibility in the curriculum; interpenetration of the various subjects and skills; emphasis on learning rather than teaching; focus on each child's thinking and problem-solving processes, and on his ability to communicate with others and freedom and responsibility for the children."

Because mathematics educators have felt the mathematics curriculum to be rather fixed, at least content-wise, they have not been at the forefront in the generation of alternatives in education. True, math educators have generally approved of and aided individualized programs, but only to effect a better climate

for traditional content teaching. There are a few glimmers of hope, though, among mathematics educators. David A. Page, of the University of Illinois Arithmetic Project located at Educational Services Incorporated, Watertown, Massachusetts, and Robert David of the Madison Project, Syracuse University, Syracuse, New York, are two mathematics educators who are significantly relating their work to the problem of alternatives in mathematics. The University of Illinois Arithmetic Project is not attempting to develop a systematic curriculum for any grade level since their determining an adequate curriculum is not possible until more alternatives exist to choose among. Page states that "it is the author's contention that more intermediate inventions are needed before really adequate mathematics programs for schools can be formulated, or even before worthwhile discussions can be held on such popular topics as the discovery method, the cognitive process, grace placement, articulation, and the objectives of mathematics curriculum in our schools."

Dr. Davis suggests the future course for his project, and in doing so, suggests the direction mathematics education and the attendant research should take for a number of years to come.

"The thing I'd like to see happen would be to get lots of experimental schools run by different people on different philosophies. Now you've got to decide, 'Okay, is that a realistic possibility or not?' It's very hard at the moment but it looks like it might happen. Right here in Ithaca there's a school that's scheduled to be closed. All the teachers in it have been transferred out to other schools and the building is to be unoccupied in September. Now one of the parents' groups wants to take it over and run it as an experimental school. They're getting quite a bit of interest and support in the community, and they'd really be starting from scratch. There would be no faculty there, no anything. They could go a long way towards building the kind of school they really want.

"I've always liked David Hawkins' remark that the problem with the 'independent variables' in education is that actually they're constants. I think the thing that hits you as you look at different schools around the United States is that they really are all pretty much alike. There <u>are</u> differences, but mostly not very big ones.

"I'd like to see <u>every</u> serious philosophy of education tried out. A particular school would select its own philosophy, develop it for all they were worth, and have the courage of their convictions when it came to implementing

it. That would mean we would have many different types of schools, some quite unlike others. Then I would let the customers choose, as they can in buying a car or choosing a restaurant.

"You name almost any theory of education you want and I'd like to see some people really go at that hammer and tongs. That's why I like the school with no building in Philadelphia. I'd like schools where attendance is voluntary. I'd like some farm schools where you could go horseback riding and do stuff like that and again maybe attendance in class was voluntary. Much of our work where we use kids is often after school, or Saturdays, or summers. That's voluntary and we've followed some of these kids for as long as five years. I think that gives you a measure of what's happening. Will the kids come on Saturdays for five years? I even like this approach in my workshop for teachers here at Ithaca. I can't tell whether these Monday evenings are really worth anything to them, but one of my criteria is going to be, will they keep coming, because really they're not getting any serious credit for it. They're mostly on the top salary level. They're not degree candidates or anything. So, will they keep coming? That gives you a lot of insight into what you're offering them.

"But the key to all of these different kinds of schools would be diversity and consumer choice. I don't rule out things like the stereotypes of the

Bereiter-Engleman approach where allegedly you have a sort of controlled, benevolent fascism or whatever you want to call it, where you really tell kids what they've got to do and you make them do it. My feeling is we just can't study these things right now because you don't find any real variations, or not very much. Every school has a little bit of this and a little bit of that."

So whither mathematics Education? Just as our space travel has simply begun with trips to the moon so math education is simply on the threshold of a new frontier.

The remaining portions of this book, then, represent some alternatives to the status quo. It is hoped that these ideas do not find their way to the "graveyard of well-written intentions-the educator's bookshelf", but rather have an effect in some of our elementary classrooms.

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"INDIVIDUAL"IZED CURRICULUM - WHAT KIND?

The American ideal of educating each and every child has continually caused the educational establishment in the United States considerable problems. Witness, as examples, the present problem of dissegregation of the public schools and the multitude of special education programs. Only recently has education been endowed with the technology with which to attack the task of individualizing instruction for all. Unfortunately, many of America's foremost educators have used this power of technology for the mundane, somewhat trivial task of "drill and practice" routines for students. This type of abuse has led many educators to feel the task of individualizing instruction has already been accomplished; at least in several of the more innovative systems. Lest there be those who would say that we have completed the task of individualized instruction, or least in theory have set the parameters and know exactly how to proceed with the necessary steps of implementation, let him note this author's spelling of "individual"ized. We have not completed the task of "individual"izing instruction and it is inconceivable that we ever will.

At the heart of instruction is the individual student - the cognitive, the affective, the psychomotor individual - all wrapped up into one throbbing mass

of individuality. Preference for the word "individual"ized instruction, as opposed to humanized instruction or personalized instruction, comes partly from the constant use of the term in educational circles over the past 10 years, but also from a recent bout with *The American College Dictionary*. This dictionary makes some interesting distinctions among the three words. According to this dictionary, "person is the most general and common word applied to human beings: the average person." With respect to the word human the dictionary states, "Human, Humane may refer to that which is, or should be, characteristic of human beings. In thus describing characteristics, Human may refer to good and bad traits of mankind alike (human kindness, human weakness), with, perhaps, more emphasis upon the latter, Human being seen then in contrast to Divine: to err is human, to forgive divine; he was only human." To complete this triumvirate, the word individual is defined as "existing as a distinct, indivisible entity" distinguished by peculiar and marked characteristics; exhibiting individuality; of which each is different or of a different design from the others." Granted that each word tends to define somewhat different characteristics of a human being, and granted also that the definitions used here were edited, it appears most useful for the educational establishment in search of an identity with the

contemporary social situation to use the phrase "individualized instruction" and to define it to include all of the other words and phrases, such as "humanized instruction", or "personalized instruction".

Individualization of mathematics instruction has also meant different things to different educators. One of the natural approaches, entrenched in many sectors of the educational community, is that of a linear, continuous program for all students. Two aspects of individual differences are taken into account with this definition: children learn at different rates and students must begin at a point in the curriculum commensurate with their previous achievement. However, this program *assumes* a common curriculum for every student, failing to take into account individual student interests, learning styles, abilities, and future goal orientations. Therefore the linear curriculum must be eliminated from our thinking if we are to get to the task of "individual"izing instruction for every student.

The educator preparing his individualized mathematics curriculum would be remiss in his planning if he simply planned a linear, continuous program for *all* students, failing to build into the curriculum a means whereby each student's personal inventory of interests, abilities, and learning styles could be exploited

to the fullest. This type of curriculum Crowder might call the "branching" curriculum, except in our "branches" we might never get back to the main tree. In other words, if one were to consider each student's mathematics experiences as a finite set, then no two sets would be equal, and quite likely, no two sets equivalent. To carry the example one step further, the number associated with the set consisting of the intersection of any two of these sets should be as close to zero as is possible.

Although the following quote is taken from a book on music education in the elementary school, it speaks to the entire scope of curriculum development for an individualized mathematics program of instruction that attempts to implement a "branching" rather than "linear" curriculum, (thereby taking advantage of the differences that do exist among children).

> Whenever a teacher meets a group of children, whatever the subject and whether the class be large or small, selected or unselected, she is confronted with the problem of individual differences, for, as Emerson once remarked 'Nature never rhymes her children nor makes two men alike.' Such variability is due to heredity and environment. Home, community, and school all leave their imprint. In every class, teachers find boys and girls who are aggressive and quiet, thoughtful and thoughtless, alert and indolent, cheerful and melancholy, sound and unsound in mind and body. Some children learn quickly, others slowly. Some seize upon opportunities to study and learn, whereas others prefer to play an instrument; some choose to listen to music and learn about compositions and composers; a few may get their greatest pleasure from

technical work and composing music; some enjoy all musical activities. Fortunately, one seldom if ever finds children who do not gain pleasure and profit from some form of the tonal art.

"The need to recognize individual differences is illustrated well by Reavis in his 'Fable of the Animal School.' Directed toward the curriculum in general, it is, however, applicable to music in particular.

> 'Once upon a time, the animals decided they must do something heroic to meet the problems of "a new world," so they organized a school. They adopted an activity curriculum consisting of running, climbing, swinging, and flying, and, to make it easier to administer, all the animals took all the subjects.
> 'The duck was excellent in swimming, better in fact than his instructor, and made passing grades in flying, but he was very poor in running. Since he was slow in running, he had to stay after school and also drop swimming to practice running. This was kept up until his web feet were badly worn and he was only average in swimming. But average was acceptable in school, so nobody worried about that except the duck.
> 'The rabbit started at the top of the class in running, but had a nervous breakdown because of so much make-up work in swimming.
> 'The squirrel was excellent in climbing until he developed frustration in the flying class, where his teacher made him start from the ground-up instead of from the treetop-down. He also developed Charley horses from overexertion and then got C in climbing and D in running.
> 'The eagle was a problem child and was disciplined severely. In the climbing class he beat all the others to the top of the tree, but insisted on using his own ways to get there.

'At the end of the year, an abnormal eel that could swim exceedingly well, and also run, climb, and fly a little, had the highest average and was valedictorian...

"Realistic efforts to care for individual differences should be reflected in the musical offerings, in courses of study, in the daily plans, and in the presentation of lessons. These provisions may be summarized as follows:

1. Differentiation in the amount and type of music, technical information, and skills for mastery in all classes.

2. Presentation of varied activities so that pupils can be guided into those which match their capabilities and interests.

3. Use of many teaching devices and techniques.

4. Encouragement and use of pupil initiative".

We in the business of teaching mathematics to children oftentimes are caught up determining what mathematics children *must* have in order to survive in some "future" existence, and fail to consider that *what* children learn may not be nearly as important as the *process* that goes along with learning mathematics. For example, what would happen if the students you are now teaching were not expected to "memorize" those fundamental facts in arithmetic. Do you think that the second graders you are now working with would find it difficult

to cope with the world of work in the 1985's and 1990's? Would mathematics instruction at the advanced levels become impossible? Might it not be that in a few years small pocket calculators will be generally available for use by students? In much the same way, and hopefully with a bit more information, the curriculum for each child has to be carefully examined. The doctor prescribing the same medicine for all his patients would lose professional standing at the very least. The mathematics teacher who instructs each of his students in the same manner should lose his teaching position. Since every student does not suffer from the same "math sickness," the teacher cannot 'treat' all the same. Likewise, teachers cannot "immunize" all students equally for something they *may* need in the unseen future. *Each child experiences his own unique world, quite different from the world of any other person that has ever lived or will ever live, and the frightening aspect of each child's uniqueness is that our knowledge of the requirements of life, both present and future, in his world is, in most cases the empty set.*

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THE RATIONALE FOR INDIVIDUALIZED INSTRUCTION

Two men look out through the bars: One sees the mud, and one the stars.

Frederick Langbridge

It is not uncommon in educational circles to read or hear the statement "Individualized instruction has long been a goal of American education". Esbensen opens his book Working With Individualized Instruction-The Duluth Experience with such a sentence. The statement is both true and false. Its truth lies in the verbosity, written and oral, that has spewed from our university and public school theorists. In practice, however, the statement holds little truth. Educators may have stated their concern for the individual, reiterated the flowery verbiage that is popular in education, but as Macbeth once said: "It is a tale told by an idiot, full of sound and fury, signifying nothing". It is time for *all* in education to put their action where their mouth is. Theory and practice must run parallel courses if educators are to do anything about making the society in which we live more palatable for all.

Until one is clear on the rationale of and the need for individualized instur-tion in the classroom it becomes extremely difficult to undertake the implementation phases that such an instructional program requires. As often happens, people do

the right thing for the wrong reasons. This problem appears to be raising its ugly head in the individualized instruction program. In one program we note that rate of progress becomes the key to instruction. In another program, student groups of assorted sizes together with the team concept permeate as the heart of the instruction. Yet it is said that individualized instruction has been, and is, the goal. Just what are we talking about? What is the reason for individualized instruction? Is the concept of individualized instruction open to variant interpretations? Similarly, is there more than one rationale for such a program of instruction?

In 1916, John Dewey wrote *"A society based on custom will utilize individual variations only up to a limit of conformity with usage; uniformity is the chief ideal within each class. A progressive society, (however), counts individual variations as precious since it finds in them the means of its own growth. Hence a democratic society must in consistency with its ideal, allow for intellectual freedom, and the play of diverse gifts and interests its educational measures."* Dewey argues for the survival and growth of a democratic society through the optimum development of individual skills and abilities. The extent of growth in a technological democratic society is directly pro-

portional, therefore, to the number of varying talents the citizenry possesses.

Dewey's logic, however, does not seem to serve as a sound basis for individualized instruction since it would be possible to train the citizenry in the needed skills by group methods and accomplish the same end goal: growth and survival of the society through the development and proliferation of individual skills and talents. And it would appear that Russia is accomplishing this growth without a program that has the individual at the heart and core of the decision-making process. Even if one grants the need of society as a basis for consideration of the individual, can we construct no sounder basis for the establishment of programs of individualized instruction?

Most educators, classroom teachers included, would rest their case for a rationale on the matter of individual differences. Skinner in his book The Technology of Teaching states that *"failure to provide for differences among students is perhaps the greatest single source of inefficiency in education"*. That students supposedly vary in the rate, extent, style, and quality of their learning was the subject of a conference held at the Learning Research and Development Center, University of Pittsburgh, Pittsburgh, Pa., April 9-10, 1965. The general question posed for the participants was: *"In what sorts of*

ways may people be expected to differ in their learning and how might these ways be measured as individual differences?" In the introduction of the report on the conference, Robert Gagne writes: *"At the present time it seems fair to say that we know considerably more about learning, its varieties and conditions, than we did ten years ago. But we do not know much more about individual differences in learning than we did thirty years ago."* Are we to justify a program of individualized instruction on a basis of ignorance? Hopefully, we can do better!

Arthur W. Melton, in summarization of the conference, shows the work that lies ahead in the examination of individual differences--"*...in fact, if psychology should be blessed with a truly great theorist in the next 20 years, his theoretical tour-de-force may well be the systematic integration of these two approaches* (information processing theories of behavior vs. S-R association theories of behavior)." Thus, it is apparent that bits and pieces are all we have to work with regarding individual differences.

Another concern with the base of individual differences as a rationalization for individualized instruction revolves around the manner in which we partition students chronologically in our public schools. Could it be possible that differences in learning are a direct result of the wrong equivalence

relation, *"is the same age as"*, used to partition our school children? If we were to define our relation on the set of all students to be *"is the same sex as"*, might we not remove some of these individual differences that plague our instruction? Might we define the relation to be *"has the same rate of learning and beginning achievement level as"*? Would this not eliminate many significant differences? Of course, had education defined such relations on students rather than the age relation for grouping purposes, it is altogether conceivable that individualized instruction would never have been needed! Or would it?

In answering that question one finds the ultimate justification for a learner-centered program of instruction. The answer revolves around the findings of the Hanover Institute and the work done there by Prof. Adelbert Ames, Jr., and his associates. In the preface to his book Education for What Is Real, Earl C. Kelley discusses the importance of these studies. *"To say that the studies are important is, in my opinion, to betray the weakness of the words. I believe that these experiments go far to supply, in a material, laboratory way, what has been lacking in our understanding of the relation between the human organism and his environment, and all that this implies--for*

education, art, diplomacy, human relations, and so on. Speaking as a teacher, I believe that if we really master these basic facts of perception, they will tell us how to arrange for the growth of children, and from this point of departure we can finally establish what we may believe about teaching and learning."

For specifics regarding these experiments, the reader should seek out a copy of Earl Kelley's book. Briefly, these experiments dealt with oddly shaped rooms, chairs, windows, and other objects which seemed to distort reality when perceived by ordinary people. Many people called these experiments *"illusions"* and of no particular import. However, men such as Dewey, Einstein, and Kelley thought otherwise. What exactly did they see for education in these experiments? Postman and Weingartner summarize the findings of the experiments and respond to their significance in the recent book Teaching As a Subversive Activity.

"What is it that Ames seemed to prove?

1. *We do not get our perceptions from the 'things' around us. Our perceptions come from us...Reality is a perception, located somewhere behind the eyes...*

2. *It seems clear from the Ames studies that what we perceive is largely a function of our previous experiences, our assumptions, and our purposes...*

3. *We are unlikely to alter our perceptions until and unless we are frustrated in our attempts to do something based on them...The ability to learn can be seen as the ability to relinquish inappropriate perceptions and to develop new-and more workable- ones.*

4. *Since our perceptions come from us and our past experience, it is obvious that <u>each individual will perceive what is 'out-there' in a unique way. We have no common world</u>..."*

How do these findings about perception seem to fit into a rationale for individualized instruction? What difference does it all make? Kelley summarizes the logic of it all when he writes: *"Now it comes about that whatever we tell the learner, he will make something that is all his own out of it, and it will be different from what we held so dear and attempted to 'transmit'. He will build it into his own scheme of things, and relate it uniquely to what he already uniquely holds as experience. Thus he builds a world all his own, and what is really important is what he makes of what we tell him, not what we intended."* Obviously, then, you <u>must</u> have a learner-centered curriculum, not because of differences in rate of learning, motivation, style of learning, or the needs of society, but because there really is no other choice. From the perception point of view, individual differences are as varied and many as the individual students in the school building. It would be foolish to attempt a program of instruction, other than individualized instruction, simply because it would be another matter of misapplication of known facts concerning perception. Children cannot afford such mismanagement.

Okay! We need a program of individualized instruction. No serious educator will argue on this matter. But what would a program look like that really does a job of individualizing instruction for every student? What kinds of behaviors would teachers and students be expected to elicit? On Nov. 18-20, 1968, the Aerospace Education Foundation and the United States Office of Education co-sponsored the National Laboratory for the Advancement of Education. An Identification Card for Individualized Instruction was developed which should aid us in consideration of the questions raised.

"An instructional system is individualized when:

1. *The characteristics of each student play a major role in the selection of objectives, sequence of study, choice of materials and procedures.*
2. *The time spent by each student in a given subject area is determined by his performance, rather than by the clock.*
3. *The progress of each student is measured by comparing his performance with his specific objectives rather than with the performance of other students.*

Students:

1. *Have available in writing the objectives toward which they are working.*
2. *Work toward a variety of objectives.*

3. *Use a variety of materials and procedures.*

4. *Move freely around the classroom.*

5. *Talk freely to each other about their work.*

6. *Pursue their objectives individually, with small groups of classmates, or with their teachers.*

Teachers:

1. *Encourage students to have a variety of objectives.*

2. *Allow students to move from place to place, based on what it takes to achieve objectives.*

3. *Spend more time answering questions of individuals and small groups than lecturing to the entire class.*

4. *Encourage students to help determine the materials they work with and the procedures they follow."*

It follows, therefore, as a final comment, that the student is the *"meaning-maker"* at the heart of the instructional process; whereas, the teacher becomes a facilitator for individual student learning. The teacher involved in such a program can see students involved in methods of inquiry, discovery, and research that were so often difficult to find in the group-oriented self-contained classroom. For the student, who has been called many things including

"nigger", there is, finally, the opportunity for some control over his own learning while in the prison of public school education.

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GOALS AND BEHAVIORAL OBJECTIVES OF SCHOOL MATHEMATICS

When a curriculum for any program of instruction is to be established, it is imperative that a philosophy or purpose of education in the local community be constructed to serve as a focus for further amplifications during curricular development. Philosophies of education have been variously stated, but most will center on one of the following:

1. Education serves to preserve and transmit the heritage of the culture.
2. Education operates as an instrument for the transformation of the culture.
3. Education seeks the welfare and development of the individual member of the culture.

It is easy to see how the philosophy directly affects curriculum development. If, for example, the chief function of education is the transmission of the "perennial truths", then one cannot help but strive for a uniform approach to teaching and learning. However, when the welfare of the individual within the culture is uppermost, creativity, problem-solving, critical thinking and the like become essential factors for education.

Education for life in this vastly complex world of the 1970's must be viewed from two points: the society's (what must it transmit to maintain itself), and the individual's (what the schools must do so that each individual might achieve self-realization and self-fulfillment). Education prior to the 1960's was primarily concerned with the preservation and transmission of the democratic way of life. This concern in the 1960's was augmented with changes in the educational process toward an individualized approach. There should be little question, considering the world, national, and local developments within the '60's, why education turned when it did toward a systematic attempt to center instruction on the individual. The need was clearly established because of a failing to deal effectively with people at the person-to-person level of the communication art.

Postman and Weingartner in their compelling book Teaching As A Subversive Activity extend the development of an "individual-centered" education. *"The new education has as its purpose the development of a new kind of person, one who-as a result of internalizing a different series of concepts-is an actively inquiring, flexible, creative, innovative, tolerant, liberal personality who can face uncertainty and ambiguity without disorientation, who can formulate*

viable new meanings to meet changes in the environment which threaten individual and mutual survival.

The new education, in sum, is new because it consists of having students use the concepts most appropriate to the world in which we all must live. All of these concepts constitute the dynamics of the questing-questioning, meaning-making process that can be called 'learning how to learn'. This comprises a posture of stability from which to deal fruitfully with change. The purpose is to help all students develop built-in, shock-proof crap detectors as basic equipment in their survival kits."

If one imagines a continuum of aims of education from society-oriented to individual-oriented, there is a movement in education on that continuum from the society end toward the individual end. It should be noted that this movement will continue for some time to come until education levels off somewhere in the middle of that continuum or until we destroy ourselves. Within the framework of individual responsibility, society needn't be too alarmed or concerned by people realizing self-goals and expectations. Members of a culture react violently only when frustrated in realizing their self-goals, while other members of the same culture are becoming rich at their expense. The general

goals of education, therefore, must adjust to the changing emphasis on the individual and his demands that he be allowed to develop to his fullest potential. This is the thrust of a program of individualized instruction.

In light of this discussion of general purposes of education in America, what about mathematics? What would be an expression of the purposes of school mathematics consistent with the individualized philosophy of education? Within what general framework must a specific program of individualized mathematics instruction be placed?

Fehr states four fundamental purposes the study of mathematics should attain. *"First, it should serve as a functional tool in solving our individual everyday problems. The questions How much? How many? What form or shape? and Can you prove it? arise every day in the lives of every citizen. People who can answer these questions with ease and accuracy are happy. Those who cannot make serious blunders and are frequently unhappy. To many people, effective citizenship and democratic action seem far removed from the effective use of basic mathematical concepts and skills. But if an effective citizen is one who earns his own living, rears and supports a family, buys and maintains his own house as a home, protects his family by adequate insurance, protects his job or*

career by proper investments, protects his old age by savings and annuities, figures and pays all his taxes honestly, with due regard to every allowable deduction, estimates and budgets his income, keeps himself informed of government finance and statistical surveys of the nation's economy, reads and interprets the business and financial section of the daily newspaper, serves in the armed forces or in the various areas for protection and preservation of our way of life, and makes thoughtful deductive analyses of a quantitative situation the predecessor and guide for his actions, then certainly efficient use of mathematics skills and concepts is very basic to democratic citizenship. It is in these life situations, or simulated situations, that the learning of mathematics is first motivated.

In the second place, mathematics serves as a handmaiden for the explanation of the quantitative situations in other subjects, such as economics, physics, navigation, finance, biology, and even the arts. The mathematics used in these areas of practice is exactly the same mathematics and involves the same mathematical concepts and skills. It is only the things to which the mathematics is applied that are different, and this is immaterial if one really understands the mathematics. This service of mathematics is exceedingly important to future

scientists, engineers, technologists, technicians, and skilled mechanics. It is a vocational or career service that is of value to large numbers of our present students.

In the third place, mathematics, when properly conceived, becomes a model for thinking, for developing scientific structure, for drawing conclusions, and for solving problems. Its postulational nature, that is, accepted relations (axioms or postulates), undefined terms, definitions, theorems, and a logic, aids all other areas of knowledge to approach scientific perfection. This same structure aids us in problem-solving methods in which we collect, o r g a n i z e, and analyze data and deduce conclusions for future action. For example, one who understands the mathematical method can easily frame the problem, 'Which is the better financially, to borrow $400 from the bank at 4% for a period of one month, or take it from my savings account paying 2%, whereby I lost the interest on the $400 for six months?' into a chain of syllogisms that leads to the correct response.

In the fourth place, mathematics is the best describer of the universe about us. In an age that has become statistical and scientific in much of its human endeavor, the need for people to understand these phenomena is not only

a cultural necessity but to some extent a necessity for intelligent action. If the mathematics of international banking cannot in some way be made understandable to the layman, how can he determine his position in voting on such matters?"

Since its publication in 1968, Goals for School Mathematics, The Report of the Cambridge Conference on School Mathematics, has unquestionably been the guiding force for developmental projects in mathematics education in the United States. What does this so-called "Cambridge Report" have to tell us regarding the goals of school mathematics?

"The subject matter which we are proposing can be roughly described by saying that a student who has worked through the full thirteen years of mathematics in grades K to 12 should have a level of training comparable to three years of top-level college training today; that is, we shall expect him to have the equivalent of two years of calculus, and one semester each of modern algebra and probability theory...

We propose to gain three years through a new organization of the subject matter and the virtually total abandonment of drill for drill's sake, replacing the unmotivated drill of classical arithmetic by problems which illustrate new mathematical concepts...

The reorganization which we refer to above has as its principal aspect the parallel development of geometry and arithmetic (or algebra in the later years) from kindergarten on...

We hope to make each student in the early grades truly familiar with the structure of the real number system and the basic ideas of geometry, both synthetic and analytic...

Moreover, we want to make students familiar with part of the global structure of mathematics. This we hope to accomplish by the 'spiral' curriculum which repeatedly returns to each topic, always expanding it and showing more connections with other topics.

On this firm foundation we believe a very solid mathematical superstructure can be erected which will make the pupils familiar with the ideas of calculus, algebra, and probability...

The conference felt that mathematics is a subject of great humanistic value: its importance to the educated man is almost as great as its importance to many technical specialists. The strongest argument for the early inclusion of the calculus was one of general education: liberal education requires the contemplation of the works of genius, and the calculus is one of the grandest edifices constructed by mankind...

Mathematics is a growing subject and all students should be made aware of this fact. This recommendation is not made merely because we feel that every educated person should know the fact, but also because the knowledge that there are unsolved problems and that they are gradually being solved puts mathematics in a new light, strips away some of its mystique, and serves to undermine the authoritarianism which has long dominated elementary teaching in this area...

Contemporary mathematical research has given us many new concepts with which to organize our mathematical thinking; it is typical of the subject that some of the most important of these are very simple. Concepts like set, function, transformation group and isomorphism can be introduced in rudimentary form to very young children, and repeatedly applied until a sophisticated comprehension is built up...

It is unquestionably possible to obscure a subject by introducing too much special terminology and symbolism; but we feel that most errors of this sort in fact cover an inadequate understanding of the subject matter. The function of language is to communicate... Special terms are good or bad exactly according to their effectiveness in communication, and the same applies to special notations and symbols...

To foster the proper attitude toward both pure and applied mathematics we recommend that each topic should be approached intuitively, indeed through as many different intuitive considerations as possible...

Another goal of our program is the inculcation of an understanding of what mathematics is (and what it is not). We need not here belabor the point that the man in the street has considerable misinformation on this point;...

Concentration on equality (=) is probably the reason why so many people are convinced that mathematics deals only with 'exact answers'...

While everyone should know about the wide range of topics suitable for mathematical analysis, it is almost equally important to understand the limitations of mathematics. The success of mathematics in one area often conjures up an inflated image of what it can do in another. It cannot solve the fundamental problems of politics, economics, or social relations...

..., it is important that each child get some experience with the more extended aspects of discussion. As the student progresses in mathematics, he will come increasingly to encounter long protracted discussions or solutions of problems. At some point in the future he will meet problems that take hours, days, or weeks for complete discussion, sometimes requiring a long sequence of lemmas or partial solutions..."

Too many educators have failed to consider the real purposes and goals of mathematics instruction, thereby "losing sight of the forest for the trees." These educators have viewed mathematics instruction as basic facts, addition or subtraction algorithms, etc., and have failed to tie together these bits and pieces for the student. With the advent of behavioral objectives and their use in programs of individualized instruction, one too often sees the achievement of hundreds of specific objectives by students even while the student population remains mathematically illiterate. This is not to imply that behavioral objectives are at fault and are not needed. Behavioral objectives become even more important when attached to broad goals that serve to direct both student and teacher to the significant mathematical learnings defined in this discussion.

Before we turn our attention to specific behavioral objectives in a mathematical program of instruction at the elementary level, consider the "Objectives for Student Growth" that have been delineated by one of the most exciting math projects in the world: The Madison Project. Even though the materials developed by the project, Robert Davis, Director, are judged by its authors to be a supplementary program in modern mathematics, the objectives of

the program, though one step away from performance criteria, represent the finest in the matter of goals of mathematics instruction. These objectives are listed here for consideration and evaluation.

"No important human activity is strictly bound by its apparent objectives; on the contrary, it goes beyond these objectives and may end up possessing values hardly contemplated at the outset. We would like to think that virtually all educational activity has this definition-defying character, and that Madison Project teaching is no exception. It may, however, be useful to consider a brief list of 'objectives' of Madison Project teaching. These objectives refer to objectives for the growth, over the years, of an individual student. The list is surely incompleted, but may prove suggestive.

(1) We want children to develop their ability to discover patterns in abstract situations.

(2) We want children to develop the kind of independent exploratory behavior that goes beyond anything the teacher suggested, that explores paths that both teacher and textbook author have overlooked, that sees open-ended possibilities for extension where others would see only closed completion of the assigned task.

(3) We want children to acquire a set of mental symbols which they can manipulate in order to 'try out' mathematical ideas. Probably all good mathematicians possess such a set of mental symbols, although they may be unable to describe them in words.

(4) We want the children to learn the really fundamental mathematical ideas, such as variable, function, graph, matrix, isomorphism, and so

on, and we want these ideas to be learned early enough in life so that they can serve as the foundation on which to build subsequent learnings.

(5) We want children to acquire a reasonable degree of mastery of important techniques.

(6) We want them to know basic mathematical facts-for example, the fact that $-1 \times -1 = +1$.

The objectives listed above are rather specific, mathematical objectives that might be described as 'cognitive'. There are also other important objectives of a more general nature.

(7) We want our students to emerge from our classes with a genuine belief that mathematics is discoverable.

(8) We want them to be able to make a realistic assessment of their own ability to discover mathematics.

(9) We hope they will genuinely recognize the open-endedness of mathematics.

(10) We hope they will develop an honest self-critical ability. This is important in mathematics, as in nearly everything else. It is no virtue to defend an incorrect line of reasoning, nor does habitual defensive action facilitate further learning.

(11) We hope our students acquire a personal commitment to the value of abstract rational analysis.

(12) We hope the students will come to value 'educated intuition'. The shrewd guess is never to be despised.

(13) We hope our students will come to feel that mathematics is 'fun' or 'exciting' or 'challenging' or 'rewarding' or 'worthwhile'.

(14) We want our students to learn something of the culture that lies behind twentieth-century man. We want them to understand mathematical history because they have lived through it. We can bring history right into the classroom: the students can live through experiences such as trying to solve $x^2 = -4$, only to find their path blocked, until finally someone makes a brilliant suggestion and they are able to move ahead. They have just witnessed a significant historical breakthrough, and they can consequently understand what this means in the history of mathematics in general. Because they have seen mathematics discovered, beheld this with their own eyes and heard it with their own ears, they can understand the process by which mathematics in general is discovered.

(15) Finally, we want our students to be able to appreciate pure mathematics for its own sake, but at the same time to be able to see mathematics in a natural relation to physics, biology, and so on."

The final step in the consideration of goals of school mathematics is the establishment of behavioral objectives. These will immeasureably aid the teacher in the implementation of an individualized program that places the student at the heart of the learning process. For behavioral objectives (performance objectives, performance criteria, student behaviors, etc.) to be useful, they must answer three related questions:

(1) What is the student to do after instruction? (How is the student to behave?)

(2) Under what conditions will the specified student performance take place?

(3) If applicable, what level of proficiency will be expected?

Examination of a behaviorally stated objective will illustrate the inclusion of these three aspects.

'(2) Given ten numerals that name whole numbers less than five hundred,
'(1) the student is able to list the nearest multiple of ten
'(3) with 90% accuracy.

What are the advantages of behaviorally stated objectives? Popham states five:

(1) A teacher or curriculum planner who specifies his objectives in terms of student behavior is able to select appropriate evaluation procedures, for there is little ambiguity with respect to the meaning of the objective.

(2) Behavioral objectives make it easy for the teacher to select suitable learning activities for the class since he knows precisely what kind of student behavior he is attempting to produce.

(3) Another advantage of behavioral objectives is that, since they are stated so specifically, the instructor himself can judge how adequate his objectives.

(4) A further significant advantage of behavioral objectives is that they can be given to the mature student in advance of the instruction so that he can focus his energies on relevant tasks. He can then avoid spending his time either in mastering peripheral material or in trying to outguess the instructor.

(5) Finally, behavioral objectives make it possible for the teacher, and others to evaluate instruction on the basis of whether the students accomplish the intended objectives. No longer need an instructor be evaluated on whether he has a 'pleasing personality' or a 'wholesome philosophy of life'. Using behavioral objectives, the teacher can chart their instructional goals and then go about accomplishing them."

To further establish the urgent need for the use of performance objectives, it becomes imperative to consider the arguments which have been directed against them. Again, Popham enumerates eleven such arguments and deftly refutes each.

"(1) Trivial learner behaviors are the easiest to operationalize, hence the really important outcomes of education will be underemphasized.

There is the danger that because of their ready translation to operational statements, teachers will tend to identify too many trivial behaviors as goals. But the very fact that the behaviors can be made explicit permits the teacher and his colleagues to scrutinize them carefully and thus eliminate them as unworthy of our educational efforts. Instead of encouraging unimportant outcomes in education, the use of explicit instructional objectives makes it possible to identify and reject these objectives which are unimportant.

(2) Prespecification of explicit goals prevents the teacher from taking advantage of instructional opportunities unexpectedly occuring in the classroom.

When one specifies explicit ends for an instructional program there is no necessary implication that the means to achieve these ends are also specified. Serendipity in the classroom is always welcome but, and here is the important point, it should always be justified in terms of its contribution to the learner's attainment of worthwhile objectives. Too often teachers believe they are capitalizing on unexpected instructional opportunities in the classroom, whereas measurement of pupil growth toward any defensible criterion would demonstrate that what has happened is merely ephemeral entertainment for the pupils, temporary diversion, or some other irrelevant classroom event.

> *(3) Besides pupil behavior changes, there are other types of educational outcomes which are important, such as changes in parental attitudes, the professional staff, community values, etc.*
>
> *(4) Measurability implies behavior which can be objectively, mechanistically measured, hence there must be something dehumanizing about the approach.*

This reason is drawn from a long history of resistence to measurement on the grounds that it must, of necessity, reduce human learners to quantifiable bits of data. This resistence probably is most strong regarding earlier forms of measurement which were almost exclusively examination-based, and were frequently multiple-choice test measures at that. But a broadened conception of evaluation suggests that there are diverse and extremely sophisticated ways of securing

qualitative as well as quantitative indices of learner performance.

> *(5) It is somehow undemocratic to plan in advance precisely how the learner should behave after instruction.*
>
> *(6) That isn't really the way teaching is; teachers rarely specify their goals in terms of measurable learner behaviors; so let's set realistic expectations of teachers.*

There is obviously a difference between identifying the status quo and applauding it. Most of us would readily concede that few teachers specify their instructional aims in terms of measurable learner behaviors; but they ought to.

> *(7) In certain subject areas, e.g., fine arts and the humanities, it is more difficult to identify measurable pupil behaviors.*

Sure it's tough. Yet, because it is difficult in certain subject fields to identify measurable pupil behaviors, those subject specialists should not be allowed to escape this responsibility. Teachers in the fields of art and music often claim that it is next to impossible to identify acceptable works of art in precise terms-but they do it all the time. In instance after instance the art teacher does make a judgment regarding the acceptability of pupil-produced artwork. What the art teacher is reluctant to do is put his evaluative criteria on the line.

(8) While loose general statements of objectives may appear worthwhile to an outsider, if most educational goals were stated precisely, they would be revealed as generally innocuous.

The unfortunate truth is that much of what is going on in the schools today is indefensible. Merely to reveal the nature of some behavior changes we are bringing about in our schools would be embarrassing. Yet, if what we are doing is trivial, educators would know it and those who support the educational institution should also know it.

(9) Measurability implies accountability; teachers might be judged on their ability to produce results in learners rather than on the many bases now used as indices of competence.

(In the April, 1970, issue of Phi Delta Kappan, guest editor Helen Bain discusses this problem of accountability in an article entitled "Self-Governance Must Come First, Then Accountability".)

This is a particularly threatening reason and serves to produce much teacher resistance to precisely stated objectives. Teachers might actually be judged on their ability to bring about desirable changes in learners. They should be,

(10) It is far more difficult to generate such precise objectives than to talk about objectives in our customarily vague terms.

(11) In evaluating the worth of instructional schemes it is often the unanticipated results which are really important, but prespecified goals may make the evaluator inattentive to the unforeseen.

Some fear that if we cleave to behaviorally stated objectives which must be specified prior to designing an instructional program, we will overlook certain outcomes of the program which were not anticipated yet which may be extremely important. They point out that some of the relatively recent 'new curricula' in the sciences have had the unanticipated effect of sharply reducing pupil enrollments in those fields. In view of the possibility of such outcomes, both unexpectedly good and bad, it is suggested that we really ought not spell out objectives in advance, but should evaluate the adequacy of the instructional program after it has been implemented.

Such reasoning, while compelling at first glance, weakens under close scrutiny. In the first place, really dramatic unanticipated outcomes cannot be overlooked by curriculum evaluators. They certainly should not be. We should judge an instructional sequence not only by whether it attains its prespecified objectives, but also by any unforeseen consequences it produces. But what can you tell the would-be curriculum evaluator regarding this problem? 'Keep your eyes open', doesn't seem to pack the desired punch. Yet, it's about all you can say. For if there is reason to believe that a particular outcome may result from an instructional sequence, it should be built into the set of objectives for the sequence."

The remainder of this section contains numerous examples of objectives of the type used in the initiation and implementation of the Duluth Individualized Programs. Each objective is related to the Content Guide of this volume. The decimal refers you to the appropriate section on the Guide and the numeral after the dash allows the teacher to number the individual Student Learning Guides used on a daily basis in the classroom for record-keeping purposes. Amplification of this point is covered in the section on Practical Aspects of Individualized Instruction.

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OBJECTIVES

1.21-001
Given ten mathematical sentences, the student is able to list the negation and its corresponding truth value of each sentence with 90% accuracy.

1.22-002
Given ten pairs of mathematical sentences, the student is able to list the disjunction and its corresponding truth value of each pair of sentences with 90% accuracy.

1.23-003
Given ten pairs of mathematical sentences, the student is able to list the conjunction and its corresponding truth value of each pair of sentences with 90% accuracy.

1.24-004
Given ten pairs of mathematical sentences, the student is able to list the conditional and its corresponding truth value of each pair of sentences with 90% accuracy.

1.25-005
Given ten pairs of mathematical sentences, the student is able to list the biconditional and its corresponding truth value of each pair of sentences with 90% accuracy.

1.2-006
Given ten sets of two or more compound sentences, the student is able to list the truth value of each set with 90% accuracy.

Sample Evaluation Items

-001
Negate the following sentence and list its truth value.
p: 4 + 2 = 7 ~p: *4 + 2 ≠ 7 true**

-002
List the disjunction of these sentences and state the associated truth value.
5+3=10; 2x8=15
5+3=10 ∨ 2x8=15 false

-003
List the conjunction of these sentences and state the associated truth value.
~(4+5=9); 7+8=8+7
~(4+5=9) ∧ 7+8=8+7 false

-004
List the conditional of these sentences and state the associated truth value.
2+3=6; 2+0=2
2+3=6 → 2+0=2 true

-005
List the biconditional of these sentences and state the associated truth value.
2+3=5; 6+0=6
2+3=5 ↔ 6+0=6 true

-006
Use truth tables to show the truth value of the following compound sentences.
(2 + 3 = 5) ∨ (6 x 7 = 56)
↔ (6 x 7 = 56) ∨ (2 + 3 = 5)
**(This compound sentence can be shown to be true whatever the truth of the constituent simple sentences.)*

2.3-007
Given the sentence "the square root of 2 is a rational number", the student is able to prove the truth value of this sentence using a proof by contradiction.

-007
Prove the truth value of the sentence:

$$r^2 = 2.$$

Although too lengthy here, the proof of falsity of this sentence depends on the unique prime factorization theorem.

1.1-008
Given ten English or mathematical equalities or inequalities, the student is able to list the equivalent mathematical or English sentence with 90% accuracy.

-008
List the equivalent English or math sentence.
The product of the sum of eight and five and the difference of ten and three is ninety-one.
(8+5) x (10-3) = 13 x 7 = 91

2.21-009
Given ten pairs of disjoint sets, n(A) < 15, the student is able to name the set that is the union of each pair within braces and list the number of each set with 90% accuracy.

-009 Name the union and number of the sets.
{Janet, Sue, Bill} = A
{Mary, Jim, Alan} = B

n(A)= 3, n(B) = 3, n(A∪B) = 6, and (A∪B) = {Janet, Sue, Bill, Mary, Jim, Alan}

2.23-010
Given five pairs of sets, n(A) < 6, the student is able to list the cross-product set and the associated multiplication sentence for each pair with 90% accuracy.

-010
List the cross-product and multiplication sentence.
$\{\Psi, \Omega, \psi\}$ =A
$\{\Sigma, \Delta\}$ =B
*$A \times B = \{(\Psi,\Sigma),(\Psi,\Delta),(\Omega,\Sigma),(\Omega,\Delta),(\psi,\Sigma),(\psi,\Delta)\}$
3 x 2 = 6*

3.11-011
Given a set whose members are thirty sets, n(A) ≤ 10, the student is able to partition the set into subsets based on the relation "is in one-to-one correspondence with" with 100% accuracy.

-011
Partition this set into subsets that are equivalent.

$$\left\{\{\chi\},\{\phi\},\{\omega\},\{\eta,\theta\},\{\delta\},\{\Omega,\psi,\Psi\},\{\xi,\pi\},\{\rho,\sigma,\tau\},\{\Upsilon,\Sigma,\Gamma\},\{\iota,\kappa,\lambda,\mu\},\{\partial,\partial\},\{\beta,\beta,\beta,\beta,\beta\}\right\}$$

The subsets are:

$\{\chi\} \approx \{\phi\} \approx \{\omega\} \approx \{\delta\}$
$\{\Omega,\psi,\Psi\} \approx \{\rho,\sigma,\tau\} \approx \{\Upsilon,\Sigma,\Gamma$
$\{\eta,\theta\} \approx \{\xi,\pi\} \approx \{\partial,\partial\}$
$\{\iota,\kappa,\lambda,\mu\}$
$\{\beta,\beta,\beta,\beta,\beta\}$

3.121-012
Given ten addition or subtraction sentences (sums < 10) and two columns marked "addend" and "sum", the student is able to list addends and sums in the appropriate columns with 90% accuracy.

-012
Place each in the proper column.

	Addend	Sum
7 - 4 = 3		
3 + 5 = 8	* 3,4	7
	3,5	8*

3.1211-013
Given ten non-empty sets, n(A) < 15, the student is able to partition each set into disjoint subsets of his choice, and list the number of each set with 90% accuracy.

-013
Partition this set into disjoint subsets and list the number of each set.

$A = \{\mu, \nu, \xi, o, \pi, \rho, \sigma, \tau, \upsilon\}$

$*B = \{\mu, \nu\}$ $C = \{\xi, o, \pi, \rho\}$ $D = \{\sigma, \tau, \upsilon\}$

n(A)= 9, n(B) = 2, n(C)=4, n(D)=3*

3.1211-014
Given ten pairs of sets, $n(A \cup B) < 10$, the student is able to list the addition sentence associated with the union of each pair of sets with 90% accuracy. ($A \cap B = \phi$)

-014
List the addition sentence.

$A = \{\rho, \sigma, \tau\}$
$B = \{\phi, \varsigma, 6, \aleph\}$
3 + 4 = 7

3.1212-015
Given ten pairs of standard numerals that name whole numbers less than 50, the student is able to list each pair of numerals in reverse order, where possible compute the sum and difference for each pair, and make a statement regarding the probable existance of a commutative property for addition and for subtraction of whole numbers.

-015
List each in reverse order, find the sum and difference for each if possible, and tell whether you think there is a CPA and a CPS for whole numbers.

48,34 *48+34=82; 34+48=82
34,48 48-34=14; 34-48= ?(no whole number here!)
I think there is a CPA, but not a CPS.*

3.1212-016
Given two pairs of numerals that name whole numbers less than 100, the student is able to justify (Using the CPA & APA) the addition algorithm for pair with 100% accuracy.

-016
Use the APA and CPA to justify 23+45=68.

*23+45= (20+3) + (40+5)
= 20+(3+40) + 5 APA
= 20+(40+3) + 5 CPA
= (20+40) + (3+5) APA
= 60 + 8
=68*

3.12122-017
Given five sets of three numerals that name whole numbers, the student is able to list 2 mathematical sentences for each showing the CPA and APA of whole numbers with 100% accuracy.

-017
List 2 sentences showing the APA and CPA
68,29,34
29+34=34+29; (68+29)+34=68+(29+34)

3.12122-018
Given ten pairs of numerals that name whole numbers less than ten, the student is able to list a mathematical sentence for each pair that shows the commutative property of addition with 100% accuracy.

3.12123-019
Given ten sets of three numerals that name whole numbers less than 10, the student is able to list a true math sentence for each set showing the APA of whole numbers with 90% accuracy.

3.12133-020
Given a manual or electric calculator, the student is able to check all arithmetical calculations for immediate knowledge of results with 100% accuracy.

3.1213-021
Given ten pairs of numerals that name whole numbers less than 10 thousand, the student is able to list the indicated difference of each pair using the short form with 90% accuracy.

3.1213-022
Given ten verbal problems involving the addition or subtraction of whole numbers less than ten thousand, the student is able to list an open mathematical sentence consistent with each problem and list the missing addend or sum with 90% accuracy.

3.1213-023
Given ten pairs of numerals that name whole numbers less than ten thousand(money form), the student is able to list the indicated sum or difference with 90% accuracy.

-018
List a sentence that shows the CPA.

3,6	4,1	8,3
*3+6=6+3	4+1=1+4	8+3=3+8*

-019
List a sentence showing the APA of whole numbers.

3,4,2	2,1,7
*(3+4)+2=3+(4+2)	(2+1)+7=2+(1+7)

-020
*(The day is not too far off when the traditional work done on algorithms will be done away with because of the greater speed and accuracy one can obtain on a calculator. This will mean more time for significant mathematics in the elementary school)

-021
List the difference.

$$\begin{array}{r} 9,056 \\ -\ 4,628 \\ \hline *4,428* \end{array}$$

because 4,428+4,628=9,056.*

-022
List an open sentence and find the missing addend or sum.
Mrs. Brown had to visit 340 more homes before her 1970 Census visitation to 1,945 home would be complete. How many homes had she already visited?
* ____ + 340 = 1,945
She had visited 1605 homes already.*

-023
List the sum or difference.

$38.65	$78.01
-24.79	+34.65
* 13.86	112.66

3.1213-024
Given ten open addition or subtraction equalities or inequalities with the set of whole numbers being the replacement set, the student is able to list the solution set for each sentence with 90% accuracy.

-024
Name the solution set using the set of whole numbers as the replacement set.
(2 x ___) + 4 > 35.
*{16,17,18,19,20,...}

3.12131-025
Given ten pairs of numerals that name whole numbers less than one thousand, the student is able to list the indicated sum or difference using the short form with renaming and 90% accuracy.

-025
List the sum or difference.
245 + 369= *614*
932 - 419= *513*

3.12131-026
Given ten pairs of numerals that name whole numbers less than one thousand, the student is able to list the sum of each pair with renaming of ones as one ten and tens as one hundred and 90% accuracy.

-026
List the sum.
445
+268
713

3.12131-027
Given ten verbal problems involving addition or subtraction of whole numbers with renaming of ones, tens, and hundreds, the student is able to list an open mathematical sentence consistent with each problem and list the missing addend or sum with 90% accuracy.

-027
List an open sentence and find the missing addend or sum.
Jimmy collected string until he had two balls, one 3,468 ft. long and the other 8,953 ft. long. How much string did Jimmy have?
*3468+8953 = ____
Jimmy had 12,421 ft. of string.

3.12131-028
Given ten pairs of numerals that name whole numbers less than one million, the student is able to list the sum of each pair using the short form with 100% accuracy.

-028
List the sum using the short form.
247,974 + 987,654=*1,235,628*

3.12131-029
Given ten pairs of numerals that name whole numbers less than one thousand, the student is able to list the sum of each pair with renaming of ones as a ten and 90% accuracy.

-029
List the sum.
346+239 = *585*

3.12131-030
Given ten pairs of numerals that name whole numbers less than one thousand, the student is able to list the sum of each pair with no renaming and 90% accuracy.

-030
List the sum.
435 + 162 = *597*

3.12131-031
Given ten verbal problems involving addition or subtraction of whole numbers with renaming of ones as a ten and ones or one ten as ten ones, the student is able to list an open math sentence consistent with each problem and list the missing sum or addend with 90% accuracy.

-031
List an open math sentence and find the missing addend or sum.
Millionaire Morris had 35 horses to give away and ended up with 17 left, How many did he give away?
35 - ___ = 17; He gave 18.

3.12131-032
Given ten pairs of numerals that name whole numbers less than one hundred, the student is able to list the sum of each pair using the short form with renaming of ones as a ten and ones with 100% accuracy.

3.12131-033
Given ten pairs of numerals that name whole numbers less than one hundred, the student is able to list the sum of each pair using an expanded form and renaming of ones as a ten and ones with 90% accuracy.

3.12131-034
Given ten verbal problems involving the addition or subtraction of whole numbers, the student is able to list an open mathematical sentence consistent with each problem and list the missing sum(less than 10) or addend with 90% accuracy.

3.12131-035
Given 20 open mathematical addition or subtraction sentences, the student is able to list the missing addend or sum (less than 20) with 90% accuracy.

3.12131-036
Given ten pairs of numerals that name whole numbers less than ten, the student is able to list the sum of each pair using the APA with 90% accuracy.

3.12131-037
Given thirty pairs of numerals that name whole numbers less than 10, the student is able to list the associated sums or differences within 3 minutes and 90% accuracy.

3.12131-038
Given ten numerals that name whole numbers less than twenty, the student is able to list the sum or difference of each number and 1 with 100% accuracy.

3.12131-039
Given twenty numerals that name whole numbers less than one hundred, the student is able to list the sum or difference of each number and 0 with 100% accuracy.

-032
List the sum using the short form.
28 + 35 = *63*

-033
List the sum using an expanded form.
27=*20 + 7
+36=*30 + 6
*50 + 13 = (50+10) +3 = 63 *

-034
List an open sentence and find the missing sum or addend.
Mother has 5 eggs. She needs 8 eggs to make a cake. How many more eggs does she need?
5 + ____ = 8; Mother needs 3 more eggs.

-035
List the missing addend or sum to make each sentence true.
9 + 6 = *15* 3 + *8* = 11 *10*-3 = 7

-036
Use the APA to find the sum of each pair of numbers.
9 + 6 = _____
*9+6 = 9 + (1 + 5)
=(9 + 1) + 5 APA
= 10 + 5
= 15

-037
List the sums or differences.
3+2=*5* 6-3=*3* 4+5=*9* 3+7=*10*

-038
List the sum or difference.
3+1=*4* 19-1=*18* 12+1=*13*

-039
List the sum or difference.
23+0=*23* 45-0=*45* 70+0=*70*

3.12131-040
Given ten pairs of numerals that name whole numbers less than one hundred, the student is able to list the sum of each pair with no renaming using the APA with 90% accuracy.

-040
List the sum using the APA.
34 + 3 = _____
*34 + 3 = (30 + 4) + 3
= 30 + (4 + 3) APA
= 30 + 7
= 37 *

3.12131-041
Given ten pairs of multiples of ten less than 100, the student is able to list the sum or difference for each pair with 90% accuracy.

-041
List the sum or difference.
40-10= *30* 50+30=*80* 90-60=*30*

3.12131-042
Given ten verbal problems involving the addition or subtraction of whole numbers, the student is able to list an open mathematical sentence for each problem consistent with the problem and list the missing addend or sum(less than 100) with no renaming and 90% accuracy.

-042
List an open sentence and find the missing addend or sum,
President Nixon used 35 pens to sign a bill into law. 14 of those pens were given to his family. How many pens were left?
35-14=_____; Nixon had 21 pens left.

3.12132-043
Given ten sets of three or more numerals that name whole numbers less than one hundred thousand, the student is able to list the sum of each set with 90% accuracy.

-043
List the sum.
42,894 + 45,010 + 23,987 = *111,891*

3.12132-044
Given ten sets of numerals that name whole numbers, the student is able to list the sum of each set with 90% accuracy.

-044
List the sum.
2,987,654
2,456
20
+ 621,697
3,611,827

3.12132-045
Given ten sets of three numerals that name whole numbers less than ten, the student is able to list the sum of each set (less than 20) with 90% accuracy.

-045
List the sum.
5+6+4= *16* 7+3+8= *18*

3.12132-046
Given ten sets of three numerals that name whole numbers less than 10, the student is able to list the sum (less than 10), of each set of numbers with 90% accuracy.

-046
List the sum.
3+2+4= *9* 3+6+0= *9*

3.1221-047
Given ten open addition or subtraction sentences using any base or numeration system, the student is able to list the equivalent addition or subtraction sentence showing the inverse relationship between addition and subtraction with 90% accuracy.

-047
List the equivalent addition or subtraction sentence.
XIV + ____ = L *L - ____ = XIV*

3.1221-048 Given ten addition or subtraction sentences (sums less than ten), the student is able to list the equivalent subtraction or addtion sentence with 90% accuracy.	-048 List the equivalent addition or subtraction sentence. 3 + 4 = 7 *7 - 4 = 3*
3.1222-049 Given ten pairs of numerals that name whole numbers, the student is able to list the indicated difference of each pair with 90% accuracy.	-049 List the difference. 56,009 - 12,574= *43,435*
3.12222-050 Given ten pairs of numerals that name whole numbers less than one thousand, the student is able to list the indicated difference of each pair with no renaming and 90% accuracy.	-050 List the difference. 956-734= *222*
3.1221-051 Given ten partitioned sets, n(A)<10, the student is able to list the subtraction sentence associated with each partitioned set with 90% accuracy.	-051 List a subtraction sentence for each partitioned set. $\{x, \psi, \omega, \int\vartheta, \phi, 5, 6\}$ *7 - 3 = 4*
3.12222-052 Given ten pairs of numerals that name whole numbers less than one hundred, the student is able to list the difference using an expanded form and no renaming of each with 90% accuracy.	-052 List the difference using an expanded form. 86*= *(80 + 6)* -34*= *(30 + 4)* *50 + 2 = 52* *
3.12223-053 Given ten pairs of numerals that name whole numbers less than one hundred, the student is able to list the difference of each pair using an expanded form and renaming of a ten as ten ones with 90% accuracy.	-053 List the difference using an expanded form. 43*= *(40 + 3)= (30 + 13)* -19*= *(10 + 9)= (10 + 9)* *20 + 4 = 24**
3.12223-054 Given ten pairs of numerals that name whole numbers less than one hundred, the student is able to list the difference of each pair with renaming using the short form and 100% accuracy.	-054 List the difference. 54 - 27 = *27*
3.12223-055 Given ten pairs of numerals that name whole numbers less than one thousand, the student is able to list the indicated difference of each pair with renaming of a ten as ten ones and 90% accuracy.	-055 List the difference. 953-838= *115*

3.12223-056
Given ten pairs of numerals that name whole numbers less than ten thousand, the student is able to list the indicated difference of each pair using an expanded form and renaming of 1 thousand as ten hundreds, 1 hundred as ten tens, and 1 ten as ten ones with 90% accuracy.

-056
List the difference using an expanded form.

6238 = (6000+200+30+8) = (5000+1100+120+18
-4359 = (4000+300+50+9) = (4000+ 300+ 50+9
1000+ 800+ 70+9
1,879*

3.12223-057
Given ten pairs of numerals that name whole numbers less than one thousand, the student is able to list the indicated difference of each pair with renaming of 1 hundred as 10 tens and 1 ten as 10 ones with 90% accuracy.

-057
List the difference.
432 - 378 = *54*

3.1231-058
Given ten arrays that picture the products of whole numbers less than ten, the student is able to list a math sentence for each array with 90% accuracy.

-058
List a multiplication sentence for each array.

X X X X X X
X X X X X X *3 x 6 = 18*
X X X X X X

3.1231-059
Given ten pairs of numerals that name whole numbers with products less than 30 and factors less than 10, the student is able to list the indicated quotient of each pair by constructing and partitioning a set into ☐ subsets or ☐ in each subset with 90% accuracy.

-059
List the quotient by constructing and partitioning a set.

24 ÷ 4 = _____
* X X X X
X X X X
X X X X *24 ÷ 4 = 6*
X X X X
X X X X
X X X X
X X X X

3.1231-060
Given ten pairs of numerals that name whole numbers less than ten, the student is able to list the product of each pair by constructing the appropriate number of equivalent disjoint sets and counting with 90% accuracy.

-060
List the product by constructing equivalent sets.

3 x 5 = ____
* X X X X X
X X X X X *3 X 5 = 15*
X X X X X

3.1232-061
Given five pairs of numerals that name whole numbers less than one hundred, the student is able to justify the multiplication algorithm to determine the product of each pair with 90% accuracy, using properties of multiplication or addition of whole numbers.

-061
JUstify the multiplication algorithm using the appropriate properties.
34x22=____
*34x22= 34x(20+2)
= 34x20 + 34x2 DPMA
=(30+4)x20 + (30+4) x 2
=30x20+4x20+30x2+4x2DPMA*

3.1232-062
Given ten addition or multiplication sentences, the student is able to list the property of multiplication or addition of whole numbers suggested by each sentence with 90% accuracy.

3.12322-063
Given ten pairs of numerals that name whole numbers less than ten, the student is able to list a multiplication sentence for each pair showing the CPM of whole numbers and prove the truth of each sentence by constructing arrays and counting with 90% accuracy.

3.12323-064
Given ten pairs of numerals that name whole numbers, each pair consisting of a multiple of 100 and a whole number less than 10, the student is able to list the product of each pair using the APM with 100% accuracy.

3.12325-065
Given 20 numerals that name whole numbers, the student is able to list the product of each number and 1 with 100% accuracy and write a general statement concerning the product of any whole number and 1.

3.12326-066
Given ten pairs of numerals that name whole number (axb=_, where 1000 < a < 10,000, and b<10), the student is able to list the product of each using the distributive property with 90% accuracy.

-062
List the property suggested by the sentence.
34 + 56 = 56 + 34 **CPA**
23x(3 + 9) = 23x3 + 23 x 9**DPMA**
456 x 1 = 456 **Identity for mult.*
(2x3)x5= 2x(3x5) **APM**

-063
List a multiplication sentence showing the CPM and prove the truth of the sentence by constructing two arrays and counting.
2,4
**2x4=4x2*
xxxx xx
xxxx xx
(8) xx
xx
(8)*

-064
List the product using the APM.
7 x 500 = ____
**7x500 = 7 x (5x100)*
= (7x5) x 100 APM
*= 35 x 100 = 3500**

-065
List the product of each and write a short sentence about the product of any whole number and 1.
34x1=**34** 5678 x 1 =**5678**
0 x 1 =**0** 12 x 1 = **12**
*The product of any whol number and one seems to be the whole number.**

-066
List the product using the DPMA.
5 x 6327=____
**5x6327= 5x(6000+300+20+7)*
=5x6000+5x300+5x20+5x7 DPMA
=30,000+1500+100+35
*= 31,635 **

3.12326-067
Given ten pairs of numerals that name whole numbers (axb=_, where 100 < a < 1000, b < 10), the student is able to list the product of each using the distributive property with 90% accuracy.

-067
List the product using the DPMA.
2 x 324=____
*2x324= 2x(300+20+4)
= 2x300+2x20+2x4 DPMA
=600+40+8
=648 *

3.12326-068
Given ten pairs of numerals that name whole numbers (axb=_, where 10 < a < 100, b < 10), the student is able to list the product of each pair using the DPMA with 90% accuracy.

-068
List the product using the DPMA,
4 x 26=___
*4 x 26= 4 x (20+6)
= 4 x 20 + 4 x 6 DPMA
= 80 + 24
=104 *

3.12326-069
Given ten pairs of numerals that name whole numbers less than ten, the student is able to list a mathematical sentence showing the distributive property of multiplication over addition and construct an array picturing the truth of the sentence of each pair with 90% accuracy.

-069
List a sentence show the DPMA and construct an array showing the truth of the sentence.
3,5
*3x(2+3) = 3 x 2 + 3 x 3 DPMA

x x | x x x
x x | x x x
x x | x x x *

3.1233-070
Given ten verbal problems involving the multiplication or division of whole number less than ten thousand, the student is able to list an open mathematical sentence consistent with each problem and list the missing factor or product with 90% accuracy.

-070
List an open sentence and find the missing factor or product.
234 planes left for Alaska with 55 people aboard each plane. How many people started the trip to Alaska?
*234x55= 12,870
12,870 people started the trip.*

3.12331-071
Given ten pairs of numerals that name whole numbers less than one million, the student is able to list the product of each pair with 90% accuracy.

-071
List the product.
12,985x37,092= *481,739,610*

3.12331-072
Given ten pairs of numerals that name whole numbers with each pair consisting of a number less than 1000 and a number less than 100, the student is able to list the product with 90% accuracy.

-072
List the product.
434 x 53=*23,002*

3.12331-073
Given ten numerals that name whole numbers less than one hundred, the student is able to list the product of each with 90% accuracy.

3.12331-074
Given ten verbal problems involving the multiplication or division of whole numbers less than ten, the student is able to list an open mathematical sentence consistent with each problem and list the missing factor or product with 90% accuracy.

3.12331-075
Given 30 pairs of numerals that name whole numbers less than 10, the student is able to list the indicated product or quotient within 3 minutes and 95% accuracy.

3.12334-076
Given ten pairs of numerals that name whole numbers where each pair consists of a multiple of 10 less than one hundred and a number less than 100, the student is able to list the product of each pair with 90% accuracy.

3.12334-077
Given ten pairs of numerals that name whole numbers where each pair consists of a multiple of 10,100,or 1000, and a number less than ten, the student is able to list the product of each pair with 90% accuracy.

3.12335-078
Given ten numerals that name whole numbers less than 100, the student is able to list the product of each number and 10 or 100 within 2 minutes and 100% accuracy.

3.1241-079
Given ten open multiplication or division sentences using any base or numeration system, the student is able to list the equivalent division or multiplication sentence showing the inverse relationship between multiplication and division with 90% accuracy.

3.1241-080
Given ten division sentences(products less than 40), the student is able to construct an array for each showing the meaning of division with 90% accuracy.

-073
List the product.
85 x 32 = **2720** _____

-074
List an open sentence and find the the missing product or factor.
Susie had 5 packages of gum. How many sticks of gum did she have?
**5 x 5 = _____; Susie had 25 sticks.*

-075
List the products or quotients.
7x8=**56*; 56÷ 8=**7* ; 1x8=**8* ; 49 ÷ 7=**7**.

-076
List the product.
20 x 43=**860** _____

-077
400 x 8=**3200** _____

-078
List the product.
45 x 10=**450** ; 29 x 100=**2900** _____

-079
List an equivalent multiplication or division sentence.
23_{five} x 36_{nine} = _____twelve
**_____twelve ÷ $^{23}five$ = $^{36}nine$*

-080
Construct an array showing the meaning of: 15 ÷ 3= 5

*

3.12421-081
Given 25 open division sentences(fundamental facts, with products less than 81), the student is able to list the missing factor or product within 2 minutes and 90% accuracy.

-081
List the missing factor or product.
24÷6=*4 ; *42 ÷6=7; 12÷*3 =4

3.12422-082
Given ten pairs of numerals that name whole numbers where each pair consists of a number less than 10,000 and a known factor less than 10, the student is able to list the indicated quotient(unknown factor) of each with remainder and 90% accuracy.

-082
List the quotient and remainder.
4266 ÷6 = *711 r. 0 .

3.12422-083
Given ten pairs of numerals that name whole numbers where each pair consists of a number less than 100 and the known factor less than 10, the student is able to list the indicated quotient and remainder for each with 90% accuracy.

-083
List the quotient and remainder.
86 ÷7=*12 r. 2

3.12422-084
Given ten pairs of numerals that name whole numbers where each pair consists of a multiple of 10, 100, or 1000, and the known factor less than 10, the student is able to list the indicated quotient (no remainder) with 90% accuracy.

-084
List the quotient.
3500 ÷7 = *500*

3.12423-085
Given ten pairs of numerals that name whole numbers, the student is able to list the indicated quotient and remainder using the estimated quotient technique based on powers of ten and multiples of those powers with 90% accuracy.

-085
List the quotient and remainder.
947÷12=

*1x12=12	60x12=720	6x12=72
10x12=120	70x12=840	7x12=84
100x12=1200	80x12=960	8x12=96
thus 10<q<100	thus 70<q<80	9x12=108

Based on this information, the quotient is(70 + 8) with a remainder the difference between (70+8)x12 and 947, that is 11.

3.12423-086
Given ten pairs of numerals that name whole numbers, the student is able to list the indicated quotient and remainder with 90% accuracy.

-086
List the quotient and remainder.
456,987÷123=*3715 r.42

3.12423-087
Given ten open division sentences involving whole number products less than 99 and factors less than 20, the student is able to, through successive subtractions, list the unknown factor for each with 100% accuracy.

3.12424-088
Given ten pairs of numerals that name whole numbers where each pair consists of the known factor a multiple of 10 less than 100 and a number less than 1000, the student is able to list the quotient and remainder with 90% accuracy.

3.1243-089
Given ten pairs of numerals that name whole numbers less than 1000, the student is able to list the indicated quotient using the right distributive property of division over addition with 90% accuracy.

3.1243-090
Given ten division sentences involving whole numbers less than 10 as factors, the student is able to list a mathematical sentence showing the right distributive property of division over addition and construct an array picturing the truth of the sentence for each with 90% accuracy.

3.131-091
The student is able to list the numeral names of the whole numbers, in order, 100 through 200 with 100% accuracy.

3.131-092
Given ten pairs of numerals that name whole numbers less than 1000, the student is able to list the symbol(<,=,>) that indicates the appropriate equality or inequality comparison of each pair with 90% accuracy.

3.131-093
Given five pairs of sets, n(A)≤10, the student is able to indicate an equivalence or non-equivalence comparison (as many as, fewer than, more than) with 100% accuracy.

-087
List the unknown factor through successive subtractions.
88÷11= *8*
*88-11=77 44-11=33
77-11=66 33-11=22
66-11=55 22-11=11
55-11=44 11-11=0

-088
List the quotient and remainder.
534÷70 = *7 r. 44*

-089
List the quotient using the RDPDA.
271÷78 = *3 r.37*
*(156 + 115)÷78 = 156÷78+ 115÷78
=(2 + 1) r.37

-090
Show the DPDA and construct an array for each.
45÷9 = 5
*(27+18)÷9= (27÷9) + 18÷9)
= 3 + 2
.....
.....
.....
.....
.....

-091
List the numbers from 100 to 200.
*100,101,102,103,104,105,106,...

-092
Use <, =, > to compare.
364*<375; 968*=967+1; 789*>234

-093
Write "as many as", "fewer than", or "more than" in the blanks provided.
Set A has *fewer than* Set B

3.131-094
Given ten sets, n(A) ≤ 10, the student is able to order the sets from fewest member to greatest number of members with 100% accuracy.

-094
Put these sets in order from fewest to greatest number of members.
A= {Ψ, τ, ξ}
B= {η, θ, Δ, α, δ}
C= {β, γ}
D= {φ, χ, ψ, ω, ϑ, ϕ, ς}
*(order) C, A, B, D.

3.131-095
The student is able to list the numeral names of whole numbers in order zero through nine using acceptable form with 100% accuracy.

-095
Write the numbers zero to nine.
1,2,3,4,5,6,7,8,9

3.131-096
Given ten pairs of numerals that name whole numbers less than ten, the student is able to list the symbol(<,=,>) that indicates the appropriate equality or inequality comparison of each pair with 90% accuracy.

-096
Compare using (<,=,>).
7 > 3; 2 < 3; 5*= 4+1

3.131-097
Given ten pairs of numerals that name whole numbers less than one hundred, the student is able to list the symbol (<,=,>) that indicates the appropriate equality or inequality comparison for each pair with 90% accuracy.

-097
Compare using (<,=,>).
54 > 12; 23 < 78; 25-12*=12+1.

3.131-098
The student is able to list the numeral names for the whole numbers zero through one hundred in order with 100% accuracy.

-098
Write the numbers 0-100.
0,1,2,3,4,5,6,7,...,56,57,58,59, 60,...,97,98,99,100.

3.131-099
Given ten set, n(A)≤10, and an ordinal(through ninth) associated with each set, the student is able to circle the member of each set named by the given ordinal with 90% accuracy.

-099
Circle the indicated member.
{K, Λ, M, N, Ξ, O}
fourth (right to left)
{υ, φ, χ, ψ, ω, ϑ, ϕ, ς, 6}
eighth

3.1411-100
Given ten numerals that name whole numbers less than 1000, the student is able to rename each number in expanded form with 100% accuracy.

-100
Rename in expanded form.
362
362= 300 + 60 + 2

3.1411-101
Given ten numerals that name whole numbers less than 100, the student is able to rename each number in the ____tens ____ones expanded form with 90% accuracy.

-101 Rename in the indicated form.
37= * 3 tens 7 ones
40= 4 tens 0 ones

3.1411-102
Given ten numerals that name whole numbers less than 100, the student is able to rename each number in expanded form with 90% accuracy.
3.1411-103
Given ten expanded or standard numerals that name whole numbers less than one million, the student is able to rename each number with the equivalent standard or expanded name with 90% accuracy.
3.1411-104
Given ten standard numerals that name whole numbers less than one million, the student is able to rename each number as a sum of products (expanded notation) with 90% accuracy.
3.1411-105
Given ten standard numerals that name whole numbers, the student is able to rename each number using the exponential form of powers of ten with 90% accuracy.
3.1412-106
Given ten sets, n(A)<10, and a list of three numeral names for each, the student is able to circle the appropriate numeral name for each set with 100% accuracy.
3.1412-107
Given ten sets, n(A)≤10, the student is able to list the numeral name associated with each set using standard acceptable numeral formation with 100% accuracy.
3.1412-108
Given the word names for the whole numbers less than 10, the student is able to list the associated numeral name for each with 100% accuracy.

3.1412-109
Given ten standard numerals that name whole numbers not greater than one million, the student is able to list the word name for each with 90% accuracy.
3.1412-110
Given ten word or numeral names for whole numbers, the student is able to rename each number using the equivalent numeral or word name with 90% accuracy.

-102
Rename in expanded form.
14=*10 + 4
78=*70 + 8*
-103
Rename in standard or expanded form
4000+800+60+7= *4867
364= 300+60+4
1,096= 1000+90+6*
-104
Rename in expanded notation.
83,196=8x10,000+3x1000+1x100+9x10+6

-105
Rename in exponential form.
357= $*3x10^2+ 5x10^1+ 7x10^0$

-106
Circle the correct numeral.

4 7 9 {Ω, χ,

-107
Write the numeral associated with each set.
$n\{\beta, \beta, \beta, \beta, \beta, \beta, \beta\} = 7$
-108
Write the correct numeral name.
five *5*
six *6*
three *3*
-109
Write the word name for each.
43,612 *forty-three thousand, six hundred twelve.*
-110
Rename in words or numerals.
Three hundred six *306*
8,957 *eight thousand nine hundre fifty-seven*

at name whole numbers less than 100,
o rename each number in expanded
y.

standard numerals that name whole
million, the student is able to
th the equivalent standard or
% accuracy.

merals that name whole numbers less
student is able to rename each
oducts (expanded notation) with 90%

merals that name whole numbers, the
name each number using the exponential
with 90% accuracy.

10, and a list of three numeral names
is able to circle the appropriate
set with 100% accuracy.

10, the student is able to list
ciated with each set using
numeral formation with 100% accuracy.

for the whole numbers less than 10,
o list the associated numeral name
curacy.

merals that name whole numbers not
lion, the student is able to list the
th 90% accuracy.

neral names for whole numbers, the
name each number using the equivalent
with 90% accuracy.

-102
Rename in expanded form.
14=*10 + 4
78=*70 + 8*

-103
Rename in standard or expanded form.
4000+800+60+7= *4867
364= 300+60+4
1,096= 1000+90+6*

-104
Rename in expanded notation.
83,196=8x10,000+3x1000+1x100+9x10+6x1

-105
Rename in exponential form.
357= $*3x10^2+ 5x10^1+ 7x10^0$

-106
Circle the correct numeral.

4 7 9 $\{\Omega, \chi, \phi, \xi\}$

-107
Write the numeral associated with each set.
$n\{\beta, \beta, \beta, \beta, \beta, \beta, \beta\}$ = 7

-108
Write the correct numeral name.
five *5*
six *6*
three *3*

-109
Write the word name for each.
43,612 *forty-three thousand, six hundred twelve.*

-110
Rename in words or numerals.
Three hundred six *306*
8,957 *eight thousand nine hundred fifty-seven*

3.1413-111
Given ten numerals that name whole numbers less than one thousand, the student is able to list the nearest multiple of ten for each with 90% accuracy.

3.1413-112
Given ten numerals that name whole numbers less than one hundred thousand, the student is able to list the nearest multiple of 100 with 90% accuracy.

3.1413-113
Given ten numerals that name whole numbers, the student is able to list the nearest indicated multiple of 10,100,1000, 10,000, or 100,000 with 90% accuracy.

3.1421-114
Given ten base-five numerals that name whole numbers, the student is able to rename each number with the equivalent base-ten numeral with 90% accuracy.

3.1421-115
The student is able to list, in order, the base-five numerals for whole numbers through one hundred twenty-six with 100% accuracy.

3.1422-116
Given ten pairs of base-five numerals that name whole numbers less than one hundred twenty-six, the student is able to list the base-five sum for each with 90% accuracy.

3.1422-117
Given ten pairs of base-five numerals that name whole numbers less than one hundred twenty-six, the student is able to list the indicated base-five difference with 90% accuracy.

3.1422-118
Given five pairs of base-five numerals that name whole numbers less than twenty-seven, the student is able to list the base-five product for each pair with 90% accuracy.

3.1431-119
Given ten Egyptian numerals or Hindu-Arabic numerals that name whole numbers, the student is able to rename each number with its equivalent Hindu-Arabic or Egyptian name with 90% accuracy.

-111
List the nearest multiple of ten.
57 **60** ; 674 **670**

-112
List the nearest multiple of 100.
5,667 **5700** ; 54,890 **54,900**

-113
List the desired multiple indicated.
234,090 **234000* (multiple/1000)
78,987 **79000** (multiple/100)
3 **0** (multiple/10)

-114
Rename in base-ten.
2041_{five} = **271 ten**

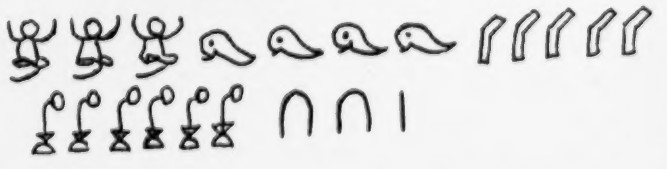
-115
List base-five numerals to 126_{ten}.
**0,1,2,3,4,10,11,12,13,14,*
20,21,22,23,24,..., 443,444,
1000,1001.

-116
List the base-five sum.
$24_{five} + 234_{five}$ = **313 five*

-117
List the base-five difference.
$213_{five} - 44_{five}$ = **114 five*

-118
List the base-five product.
$12_{five} \times 34_{five}$ = **1013 five*

-119
Rename in Hindu-Arabic or Egyptian numerals.
3,456,021



3.1432-120
Given ten base-ten or Roman numerals that name whole numbers less than 2000, the student is able to rename each number with its equivalent Roman or base-ten numeral name with 90% accuracy.

3.22125-121
Given ten numerals that name integers, the student is able to list the additive inverse of each with 90% accuracy.

3.2213-122
Given ten pairs of numerals that name integers, the student is able to list the sum of each pair with 90% accuracy.

3.2223-123
Given ten pairs of numerals that name integers, the student is able to list the indicated difference with 90% accuracy.

3.2233-124
Given ten pairs of numerals that name integers, the student is able to list the product of each pair with 90% accuracy.

3.2242-125
Given ten pairs of numerals that name integers, the student is able to list the indicated quotient with 90% accuracy.

3.31-126
Given ten fractions that name rational numbers of arithmetic, the student is able to construct number rays for each and associate each fraction with a specific point on the ray with 90% accuracy.

3.31-127
Given ten fractions that name rational numbers between zero and one, the student is able to partition a set or region to show the meaning of each fraction with 90% accuracy.

-120
Rename using base-ten or Roman numerals.
MCMLXX *1970*

-121
Name the additive inverse of each.
+34 *-34* ; -100 *+100* ; -4 *+4*

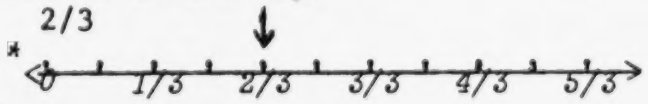
-122
List the sum.
+345 + -675= *-330*

-123
List the difference.
-1253 - -589= *-664*

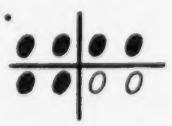
-124
List the product.
-12x+36= *-432*

-125
List the quotient.
+4644 ÷ -54= *-86*

-126
Associate each fraction with a point on the number ray.



-127
Show the meaning of 3/4 by partitioning a set.



3.31-128
Given ten regions and a unit fraction for each, the student is able to show the unit fractional part of the region by drawing with 100% accuracy.

3.3212-129
Given ten open mathematical sentences involving the addition and multiplication of rational numbers of arithmetic, the student is able to name the property of addition or multiplication of rational numbers of arithmetic suggested by each sentence with 100% accuracy.

3.32122-130
Given ten open mathematical sentences involving the addition of rational numbers, the student is able to list the missing addend or sum using the CPA or APA of rational numbers of arithmetic with 90% accuracy.

3.32122-131
Given ten pairs of fractions that name rational numbers of arithmetic, the student is able to list an addition sentence for each pair that shows the CPA of rational numbers of arithmetic and prove the truth of each sentence by adding with 90% accuracy.

3.32123-132
Given ten sets of three fractions that name rational numbers of arithmetic, the student is able to list an addition sentence for each set that shows the APA of rational numbers of arithmetic and prove the truth of each sentence by adding with 90% accuracy.

3.32131-133
Given ten pairs of like fractions(a/n,b/n) that name rational numbers of arithmetic, the student is able to list the sum of each pair in simplest form with 90% accuracy.

3.32132-134
Given ten pairs of unlike fractions(a/n,b/m) that name rational numbers of arithmetic, the student is able to list the sum of each pair in simplest form and mixed form, if appropriate, with 90% accuracy.

-128
Color 1/5 of this region.
*

-129
Name the property suggested by each.
(1/2+3/4)+5/6= 1/2+(3/4+5/6) *APA*
1/1+2/2=2/2+1/1 *CPA*
3/4x1/1=3/4 **Identity for Mult.*
1/2+0/1=1/2 **Identity for Add.*

-130
List the missing sum or addend.
1/2 +*3/4 = 3/4 + 1/2

-131
List a sentence showing the CPA and prove each by adding, 2/7&5/6.
*2/7 + 5/6 = 5/6 + 2/7
2/7+5/6= 47/42
5/6+2/7= 47/42

-132
List a sentence showing the APA and prove each by adding, 1/2,1/3,1/4.
**(1/2+1/3)+1/4 = 1/2+(1/3+1/4)*
5/6 + 1/4 = 13/12
1/2 + 7/12= 13/12

-133
List the sum in simplest form.
2/3, 1/3
**2/3+1/3 = 3/3 = 1/1*

-134
List the sum.
4/5, 1/2
**4/5 + 1/2 = 8/10 + 5/10 = 13/10*
*= 1 3/10**

3.32132-135
Given ten pairs of fractions that name rational numbers of arithmetic, the student is able to list the sum of each pair in simplest form with 90% accuracy.

-135
List the sum in simplest form.
23/35, 6/7
**23/35 + 6/7 = 161/245 +210/245*
= 371/245
*= 1 126/245**

3.32133-136
Given ten pairs of decimals that name rational numbers of arithmetic(whose least place is hundredths),the student is able to list the sum of each pair with 90% accuracy.

-136
List the sum.

```
  246.45
+9648.29
--------
*9894.74*
```

3.32233-137
Given ten pairs of decimals that name rational numbers of arithmetic(whose least place is hundredths),the student is able to list the indicated difference for each pair with 90% accuracy.

-137
List the difference.

```
  768.04
 -345.76
--------
*422.28*
```

3.32133-138
Given ten pairs of decimals that name rational numbers of arithmetic, the student is able to list the indicated sum or difference of each pair with 90% accuracy.

-138
List the sum or difference.

```
  1.9754          3.0961
+23.4567        - 0.8765
--------        --------
*25.4321*       *2.2196*
```

3.32133-139
Given ten verbal problems involving the addition or subtraction of decimals whose least place is hundredths, the student is able to list an open mathematical sentence consistent with each problem, and list the missing addend or sum of each with 90% accuracy.

-139
List an open sentence and find the missing addend or sum.
Mr. Smart had 34.23 square yards of metal to make a rocket. He used 29.67 square yards in construction. How many square yards remained?
** ____ +29.67 = 34.23*
Mr. Smart had 4.56 square yards left.

3.32133-140
Given ten verbal problems involving the combination of any two operations of addition, subtraction, multiplication, or division of decimals that name rational numbers of arithmetic, the student is able to list an open math sentence consistent with each problem and list the missing addend, sum, factor, or product with 90% accuracy.

-140
List an open sentence and find the missing sum, addend, factor, or product.
The Chrysler Motor Co. had 5 cars that weighed 5679.872 lbs.per car. The Ford Motor Co. had 12 cars that weighed 4259.902 lbs. per car. These cars were to be shipped to Germany. The boat had space for 79000 lbs. How many cars were able to be shipped?
**CMC weight=28399.360 ; FMC weight=51118.824*
Total=79518.184;thus 16 cars went.

3.32134-141
Given ten pairs of mixed numerals that name rational numbers of arithmetic, the student is able to list the sum of each pair in mixed and simplest form with 90% accuracy.

3.32231-142
Given ten pairs of like fractions (a/n,b/n) that name rational numbers of arithmetic, the student is able to list the indicated difference in simplest form with 90% accuracy.

3.32232-143
Given ten pairs of fractions that name rational numbers of arithmetic, the student is able to list the indicated difference in simplest form with 90% accuracy.

3.32232-144
Given ten pairs of unlike fractions(a/n,b/m) that name rational numbers of arithmetic, the student is able to list the indicated difference of each pair in simplest form with 90% accuracy.

3.3231-145
Given five pairs of fractions that name rational numbers of arithmetic less than one, the student is able to construct a rectangular region showing the multiplication of the fractions and list the product of each with 90% accuracy.

3.32326-146
Given ten fractions or mixed numerals that name rational numbers of arithmetic, the student is able to list the reciprocal (multiplicative inverse) of each with 90% accuracy.

3.3233-147
Given ten verbal problems involving the multiplication of rational numbers of arithmetic, the student is able to list an open mathematical sentence consistent with each problem, and list the unknown factor or product with 90% accuracy.

-141
List the sum in simplest form.

45 3/4
+ 9 5/6
55 7/12

-142
List the difference in simplest form.

3/4, 1/4

3/4 - 1/4 = 2/4 = 1/2

-143
List the difference in simplest form.

7/8, 1/3

7/8-1/3 = 21/24-8/24=13/24

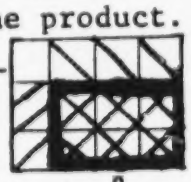
-144
List the difference in simplest form.

3/5-1/2

3/5-1/2= 6/10-5/10=1/10

-145
Construct a region to picture the multiplication and list the product.

2/3 x 3/4= *1/2*



-146
List the reciprocal of each.

5/6*1 1/5* ; 3 1/2*2/7*

-147
List an open math sentence and find the missing factor or product.

Mrs. Bloom wanted to split a piece of cloth 5 3/8 ft. in length into 3 pieces. But before she would do that she decided that she needed twice as much cloth. How much cloth did she need?

2 x 5 3/8 = _____

She needed 10 3/4 ft. of material.

3.32331-148
Given ten pairs of fractions that name rational numbers of arithmetic, the student is able to list the product of each pair in simplest form with 90% accuracy.

-148
List the product in simplest form.
5/8, 7/11
5/8x7/11= 35/88

3.32332-149
Given ten pairs of decimals that name rational numbers of arithmetic(least place ten thousandths), the student is able to list the product of each with 90% accuracy.

-149
List the product.
4.291, 0.025
4.291x0.025=0.107305

3.32333-150
Given ten pairs of mixed numerals that name rational numbers of arithmetic, the student is able to list the product of each pair in mixed and simplest forms with 90% accuracy.

-150
List the product.
3 1/2, 5 2/3
3 1/2x5 2/3 = 19 5/6

3.3243-151
Given ten pairs of fractions or mixed numerals that name rational numbers of arithmetic, the student is able to list the indicated quotient of each pair in simplest form with 90% accuracy.

-151
List the quotient in simplest form.
5/8 , 3/4
5/8 ÷ 3/4 = 5/6

3.32432-152
Given ten pairs of decimals whose least place is thousandths that name rational numbers of arithmetic, the student is able to list the indicated quotient for each to the nearest indicated tenth, hundredth, or thousandth with 90% accuracy.

-152
List the quotient to the nearest tenth.
52.176 ,3.4
52.176 ÷ 3.4 = 15.3

3.331-153
Given five pairs of unit fractions, the student is able to list the symbol (<,=,>) that indicates the appropriate equality or inequality comparison for each pair with 90% accuracy.

-153
List the appropriate symbol (<,=,>).
1/5 __<__ 1/4; 1/3 __>__ 1/5.

3.331-154
Given ten pairs of fractions that name rational numbers of arithmetic between 0 and 1, the student is able to list the symbol (<,=,>) that indicates the indicated equality or inequality comparison of each pair with 90% accuracy.

-154
List the apprppriate symbol (<,=,>).
2/3 __<__ 3/4; 1/2+1/4*= __3/4; 7/8 __>__ 1/2.

3.331-155
Given ten sets of decimals that name rational numbers of arithmetic, the student is able to list each set in order from least to greatest with 90% accuracy.

-155
List in order from least to greatest.
12.045, 11.987, 13.001, 12.145, 11.050
11.050, 11.987, 12.045, 12.145, 13.001

3.33-156
Given ten sets of mixed numerals or fractions that name rational numbers of arithmetic, the student is able to arrange each set in order from least to greatest with 90% accuracy.

-156
List in order from least to greatest.
3 1/4, 14/4, 25/8
25/8, 3 1/4, 14/4

3.33-157
Given ten pairs of decimals that name rational numbers of arithmetic (least place is hundredths), the student is able to list the symbol (<,=,>) to indicate the appropriate equality or inequality comparison with 90% accuracy.

-157
List the symbol (<,=,>) that is required.
23.06 < 23.16; 0.98 > 0.97

3.34-158
Given ten decimals that name rational numbers of arithmetic (least place hundredths), the student is able to rename each with the equivalent word name and 90% accuracy.

-158
Rename in word form.
156.09 *one hundred fifty-six and nine hundredths*

3.34-159
Given ten decimals that name rational numbers of arithmetic, the student is able to list the word name for each with 100% accuracy.

-158
Rename in word form.
4.06792
*four and six thousand seven hundred ninety-two hundred thousandths.

3.341-160
Given ten mixed numerals or fractions that name rational numbers of arithmetic, the student is able to rename each number using the equivalent fraction or mixed form with 90% accuracy.

-160
Rename in fraction or mixed form.
23 1/4= *93/4*
78/9= *8 2/3*

3.341-161
Given ten fractions that name rational numbers of arithmetic, the student is able to name the simplest form of the number with 90% accuracy.

-161
Rename in simplest form.
14/28 =*1/2*

3.341-162
Given ten fractions that name rational numbers of arithmetic, the student is able to list the set of equivalent fractions for each number with 90% accuracy.

-162
List the set of equivalent fractions.
3/4
*{3/4,6/8,9/12,12/16,15/20,...}

3.342-163
Given ten fractions that name rational numbers of arithmetic greater than one, the student is able to rename each in mixed numeral form with 90% accuracy.

-163
Rename in mixed form.
197/5 =*39 2/5*

3.342-164
Given ten fractions that name rational numbers of arithmetic, the student is able to list an equality or inequality sentence for each comparing each number with the fraction 1/1 with 90% accuracy.

-164
List an equality or inequality sentence comparing each fraction with 1/1.
11/12 23/22
*1/1 > 11/12 1/1 < 23/22

3.342-165
Given ten decimals that name rational numbers(least place is hundredths), the student is able to rename each as a mixed numeral with 90% accuracy.

3.343-166
Given ten fractions that name rational numbers, the student is able to rename each in terminating, non-repeating decimal form with 90% accuracy.

3.343-167
Given ten terminating, non-repeating decimals that name rational numbers of arithmetic, the student is able to rename each decimal in simplest fraction form with 90% accuracy.

3.343-168
Given ten fractions that name rational numbers of arithmetic less than one (excluding the non-terminating, non-repeating form), the student is able to rename each fraction in decimal form with 90% accuracy.

3.3451.169
Given ten fractions or numerals in per cent form that name rational numbers of arithmetic, the student is able to rename each number using the equivalent per cent or fraction form with 90% accuracy.

3.3452-170
Given ten open mathematical sentences involving proportion, the student is able to list the missing member (means or extremes) of the proportion with 90% accuracy.

3.3453-171
Given ten verbal problems involving per cent, the student is able to list an open mathematical sentence consistent with each problem based on equivalent ratios and list the missing member with 90% accuracy.

3.346-172
Given ten decimals that name rational numbers of arithmetic, the student is able to rename each decimal using an expanded form with 90% accuracy.

-165
Rename each as a mixed numeral.
23.172
**23 172/1000*

-166
Rename in terminating, non-repeating form.
1/8 =**0.125**

-167
Rename in simplest fraction form.
0.468
167/250

-168
Rename in decimal form.
1/7
**0.$\overline{142857}$*

-169
Rename in equivalent fraction or per cent form.
1/2 **50%** 68% **17/25**

-170
List the missing member.
$\frac{50}{100} = \frac{(*2*)}{4}$

-171
List an open math sentence and find the missing member.
John received a score of 24 right out of a total of 25 items on the test. What was his per cent score?
x/100 = 24/25
**Therefore John's per cent was 96%.*

-172
Rename using an expanded form.
23.709
23.709=20+3+7/10+9/1000

3.346-173
Given ten decimals that name rational numbers of arithmetic(least place hundredths), the student is able to rename each in expanded form with 90% accuracy.

3.4211-174
Given a finite system, S_n, where n<10, the student is able to construct an addition table and a multiplication for that system with 100% accuracy.

3.4212-175
Given a finite system, S_n, where n < 10 and n is prime, the student is able to verify all cases of the properties of addition in that system including closure, commutative, associative, identity, and inverse properties with 90% accuracy.

3.4222-176
Given a finite system, S_n, where n < 10, and n is a prime, the student is able to verify all cases of the properties of multiplication in that system including closure, commutative property, associative, identity, inverse, and distributive properties with 90% accuracy.

3.431-177
Given ten numerals that name whole numbers, the student is able to name the numbers that are divisible by 2, 3, 5, or 9 within 3 minutes and 90% accuracy.

-173
Rename in expanded form.
8.45
8.45= 8+4/10+5/100

-174
Construct an addition and a multiplication table for finite system, S_3.

*

+	0	1	2	x	0	1	2
0	0	1	2	0	0	0	0
1	1	2	0	1	0	1	2
2	2	0	1	2	0	2	1

*

-175
Verify all cases of the CPA, APA, Closure, identity, and inverse properties of S_3.
The verification of all of the cases of these properties is too lengthy to be printed here, but would include all possible combinations of each property based on the table in -174. For example, the CPA would require verification of sentences as 0+1=1+0;0+2=2+0;1+1=1+1;1+2=2+1, and so on.

-176
Verify all cases of the CPM, APM, closure, identity, inverse, and distributive properties of S_3.
**Again, too lengthy, but try it for yourself. The possibilities in S_3 are much more limited than in other finite systems.*

-177
Which numbers are divisible by 2? by 3? by 5? or by 9?
99864, 12402,4803,532,864,2100, 5285, 8690
**by 2-99864,12402,532,864,2100,8690*
by 3-99864,12402,4803,864,2100 .
by 5-2100,5285,8690
by 9-864,12402, 99864

4.1-178
The student is able to list the even numbers less than 10 and construct a set that shows that each is an even number with 100% accuracy.

4.1-179
The student is able to list the odd numbers less than ten with a set showing that each number is odd with 90% accuracy.

4.1-180
Given ten pairs of standard numerals that name whole numbers, the student, without adding or subtracting, is able to state the evenness or oddness of the indicated sum or difference of each pair with 100% accuracy in one minute.

4.2-181
Given ten numerals that name whole numbers less than 81, the student is able to list the set of factors of each with 90% accuracy.

4.2-182
Given ten numerals that name whole numbers, the student is able to list the set of factors of each number with 90% accuracy.

4.2-183
Given ten numerals that name whole numbers, the student is able to underline those numerals that name prime numbers, and list the set of factors of the composite numbers with 90% accuracy.

4.4-184
The student is able to list the multiples of 100 through 1000 with 100% accuracy.

-178
List the even numbers to 10 and draw a picture to show that each is even.
2,4,6,8,10
xx xx xxxx xxxxx xxxxxx
(2) xx xxxx xxxxx xxxxxx
(4) (6) (8) (10) *

-179
List the odd numbers to 10 and draw a picture to show that each is odd.
* 1,3,5,7,9*
x xx xxx xxxx xxxxx
x xx xxx xxxx
(1) (3) (5) (7) (9)

-180
List O for oddness or E for evenness of the sum or difference.
356+341=____; 729-738=____
O *O*

-181
List the set of factors of each.
56
{1,2,4,7,8,14,28,56}

-182
List the set of factors of each.
340
{1, 2, 4,5,10,17,20,34,68,85, 170,340}

-183
Underline the primes and list the set of factors of the composites.
7,28
{1,2,4,7,14,28}

-184
List the multiples of 100 to 1000.
100,200,300,400,500,600,700,800, 900,1000

4.4-185
The student is able to list the multiples of 10 through one thousand with 100% accuracy.

-185
List the multiples of 10 to one thousand.
*10,20,30,40,50,60,70,...,
560,570,580,590,600,...,
980,990,1000*

4.4-186
The student is able to list the numeral names for any sequence of multiples of 5 with 100% accuracy. (at least 15 members in the sequence.)

-186
List any 15 member sequence of multiples of 5.
*135,140,145,150,155,160,165,170,
175,180,185,190,195,200,205,*

4.4-187
Given ten numerals that name whole numbers less than 100, the student is able to list the indicated set of multiples of each with 90% accuracy.

-187
List the set of multiples of 21 less than 500.
*21,42,63,84,105,126,147,...,
420, 441, 462,483*

4.5-188
Given ten fractions that name rational numbers of arithmetic less than 1, the student is able to list the greatest common factor (GCF) of each numerator and denominator and name the equivalent fraction in simplest form with 90% accuracy.

-188
List the GCF for each numerator and denominator and rename each fraction in simplest form.
12/18 15/65
*GCF of 12&18 is 6. *GCF of 15&65
12/18=2/3* is 5.
15/65=3/13*

4.5-189
Given ten pairs of numerals that name whole numbers less than one hundred, the student is able to list the greatest common factor of each pair with 90% accuracy.

-189
List the GCF for each pair.
14,42
GCF is 14

4.5-190
Given ten pairs of numerals that name whole numbers less than 1000, the student is able to list the greatest common factor of each pair with 90% accuracy.

-190
List the GCF of each.
132, 456
GCF is 12

4.6-191
Given ten pairs of fractions that name rational numbers of arithmetic, the student is able to name the least common denominator of each pair with 90% accuracy.

-191
List the LCD of each.
6/7, 3/4
LCD is 28

4.6-192
Given ten pairs of numerals that name whole numbers, the student is able to list the least common multiple of each with 90% accuracy.

-192
List the LCM of each.
25,36
LCM is 900

4.7-193
Given ten numerals that name whole numbers less than 1000, the student is able to list the prime factorization of each with 90% accuracy.

4.94-194
Given ten numerals that name whole numbers, the student is able to list the square of each with 90% accuracy.

4.95-195
Given ten factorial numerals that name whole numbers, the student is able to rename each in standard numeral form with 90% accuracy.

5.11-196
Given the words "point, ray, angle, and segment", the student is able to construct and name two pictures of each word with 100% accuracy.

5.11-197
Given a set of ten labelled pictures of geometric figures (point, segment, ray, line, and angle), the student is able to partition the set into subsets based on the relation "is the same type of geometric figure as" and name each subset by the word name associated with those figures with 90% accuracy.

5.11-198
The student is able to make a drawing that utilizes a minimum of two segments, two lines, two rays, and two angles.

5.11-199
The student is able to make a drawing that utilizes pictures of circles, squares, rectangles, quadrilaterals, trapezoids, and polygons.

5.11811-200
Given a set of 15 geometric figures, the student is able to place an X on all simple closed curves with 100% accuracy.

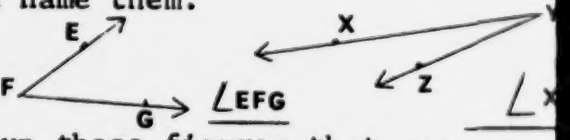
-193
List the prime factorization.
568
2x2x2x71

-194
List the square of each.
24, 5,122

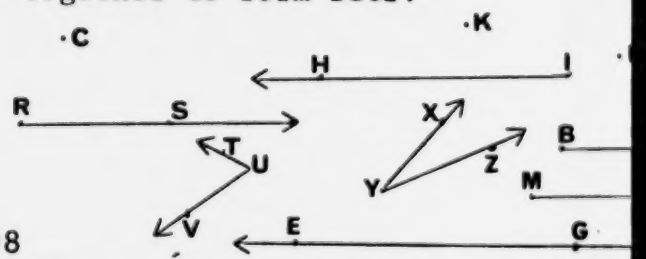
24^2= 576; 5^2=25; 122^2= 14884

-195
Rename in standard form.
8! = 8x7x6x5x4x3x2x1= 40320

-196
Draw two pictures of an angle and name them.



-197
Group those figures that are the same type of geometric figures together to form sets.



-198
Draw a picture that uses two segments, two lines, two rays, and two angles.
(We'll leave this to the student's creative abilities.)

-199
Draw a picture that uses circles, squares, rectangles, quadrilaterals, trapezoids, and polygons.
(Again, the student has wide latitude in what he draws)

-200
Place an X on all simple closed curves.

s that name whole numbers less than 1000,
le to list the prime factorization of each
.

s that name whole numbers, the student is
square of each with 90% accuracy.

al numerals that name whole numbers, the
o rename each in standard numeral form
.

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o construct and name two pictures of each
curacy.

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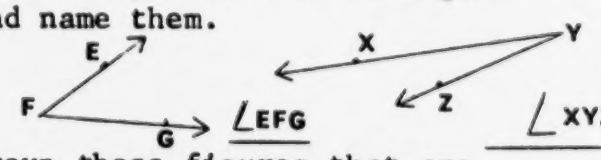

-197
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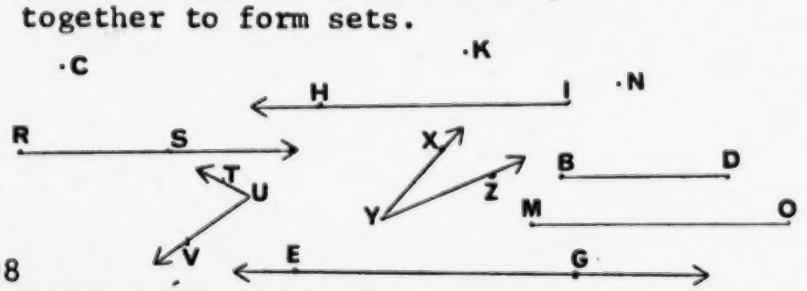


-198
Draw a picture that uses two segments, two lines, two rays, and two angles.
(We'll leave this to the student's creative abilities.)

-199
Draw a picture that uses circles, squares, rectangles, quadrilaterals trapezoids, and polygons.
(Again, the student has wide latitude in what he draws)

-200
Place an X on all simple closed curves.

5.1181-201
Given ten simple closed curves, the student is able to construct a region congruent to the interior of each curve with 90% accuracy. (tracing)

5.11811-202
The student is able to draw pictures of ten curves and identify which are simple closed curves and which are not with 100% accuracy.

5.1182-203
Given a set of 15 geometric figures, the student is able to identify each figure by placing the letter C(circle), T(triangle), R(rectangle), orS(square) in the interior of each figure with 100% accuracy.

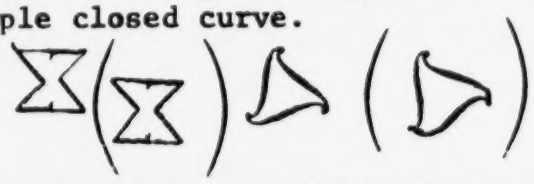
5.11822-204
Given twelve quadrilaterals, the student is able to name each as a trapezoid, parallelogram, rectangle, or square (as many as apply) with 90% accuracy.

5.115-205
Given ten angles, the student is able to name each as a right, obtuse, or acute angle with 100% accuracy.

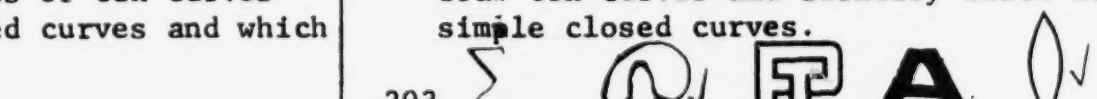
5.116-206
Given a picture of ten lines, some intersecting and some parallel, the student is able to name the pairs of parallel lines with 90% accuracy.

5.1102-207
Given a compass and the measures of the radii of five circles, the student is able to construct five circles based on the given measures with 90% accuracy.

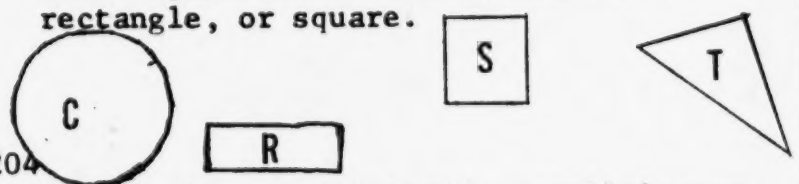
-201
Construct a figure congruent to eahh simple closed curve.



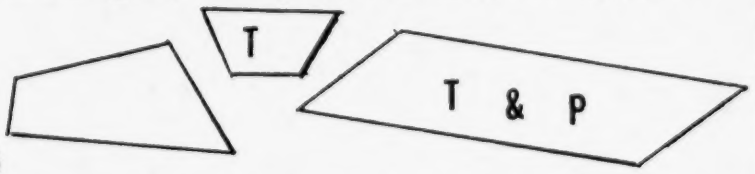
-202
Draw ten curves and identify which are simple closed curves.



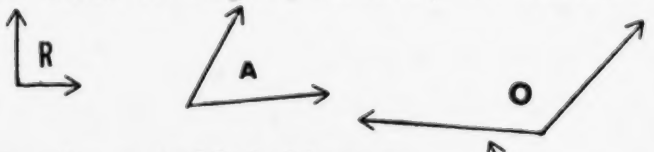
-203
Place C,T,R or S in the interior of each figure for a circle, triangle, rectangle, or square.



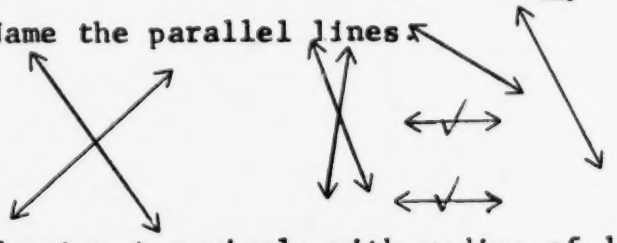
-204
Name each as a trapezoid, parallelogram, rectangle, or square(as many as apply).



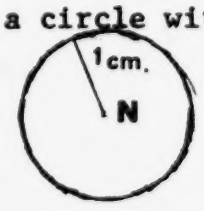
-205
Name each as right, obtuse, or acute.



-206
Name the parallel lines.



-207
Construct a circle with radius of 1cm.



5.221-208
Given a compass and straightedge, the student is able to construct five concentric circles and list the measure of the radius and diameter of each circle with 90% accuracy.

5.1102-209
Given a segment, an angle, a triangle, a rectangle, and a compass, the student will construct a geometric figure congruent to each with 100% accuracy.

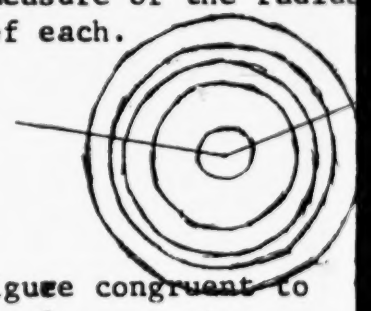
5.1102-210
Given a unit segment and ten other segments, the student is able to list the measure of each segment based on the unit segment to the nearest ½ unit with 90% accuracy.

5.1102-211
Given two segments and a point associated with each not on the segment, the student is able to construct a perpendicular from the associated point to the given segment with 100% accuracy.

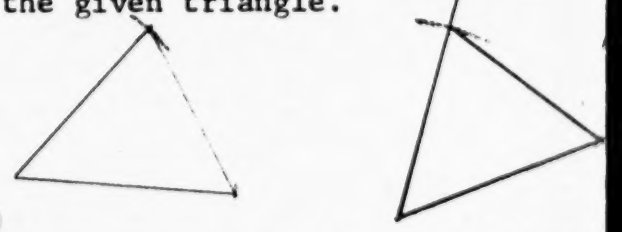
5.1102-212
Given five segments, a compass, and a straightedge, the student is able to construct the perpendicular bisector of each with 100% accuracy.

-208
Construct five concentric circles and list the measure of the radius and diameter of each.

radii - 3mm.
8mm.
1cm.
1.3cm.
2.1 cm.



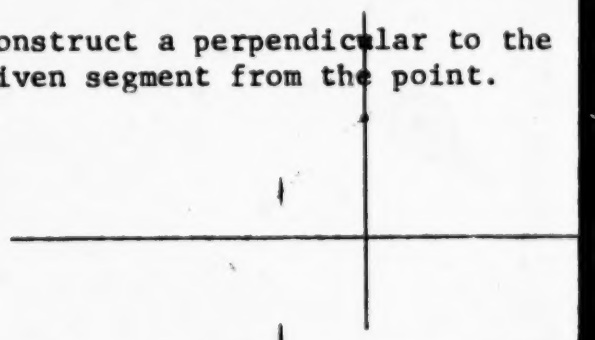
-209
Construct a figure congruent to the given triangle.



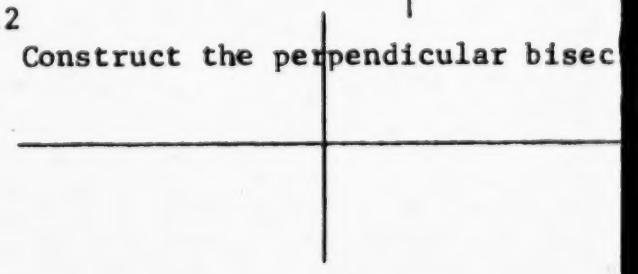
-210
List the measure of each segment based on the unit segment.

The segment is 3 units l

-211
Construct a perpendicular to the given segment from the point.



-212
Construct the perpendicular bisec



s and straightedge, the student is able to
concentric circles and list the measure
and diameter of each circle with 90% accuracy.

t, an angle, a triangle, a rectangle, and a
tudent will construct a geometric figure
ach with 100% accuracy.

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t the measure of each segment based on the
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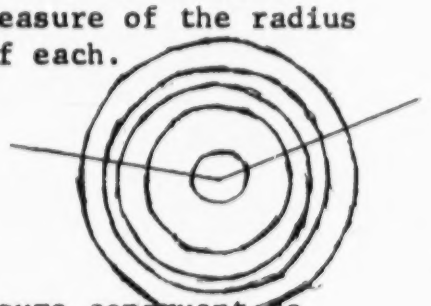
, the student is able to construct a
from the associated point to the given
00% accuracy.

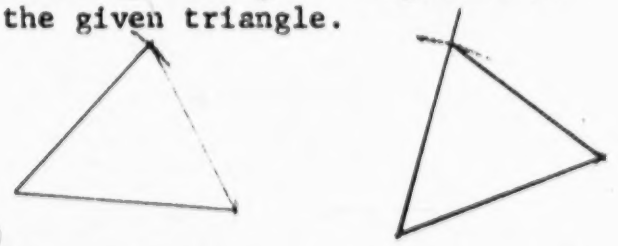
ments, a compass, and a straightedge, the
e to construct the perpendicular bisector
00% accuracy.

-208
Construct five concentric circles and list the measure of the radius and diameter of each.



radii - 3mm.
8mm.
1cm.
1.3cm.
2.1 cm.

-209
Construct a figure congruent to the given triangle.

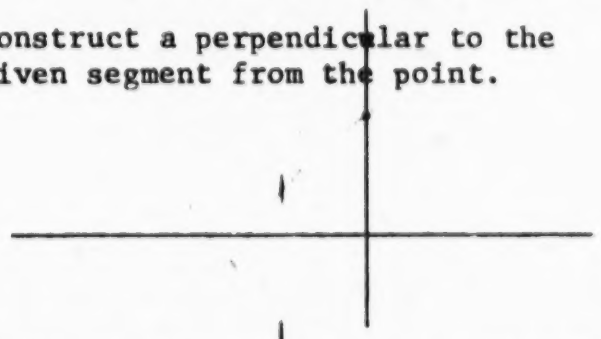


-210
List the measure of each segment based on the unit segment.

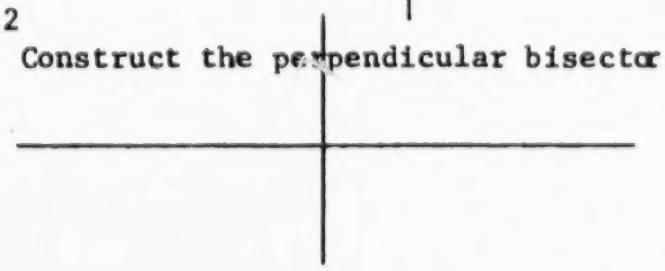
_____
(unit)

The segment is _3_ units long

-211
Construct a perpendicular to the given segment from the point.

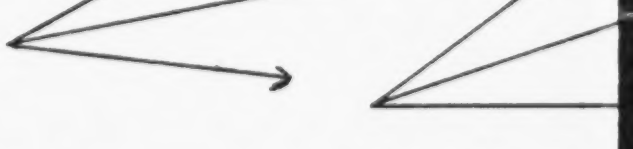


-212
Construct the perpendicular bisector



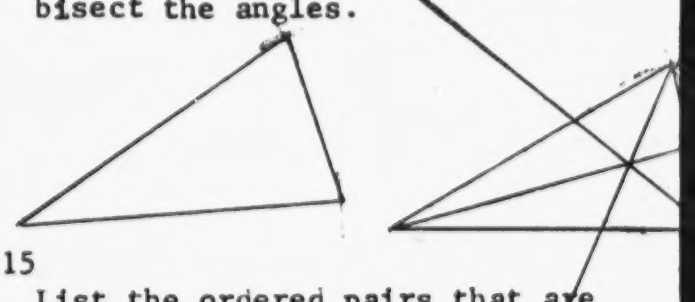
5.1102-213
Given five angles, a compass, and a straightedge, the student is able to construct angles congruent to the' given angles, and the angle bisectors of each with 90% accuracy.

-213
Construct a congruent angle and bisector.



5.1102-214
Given two triangles, a compass, and a straightedge, the student is able to construct triangles congruent to the given triangles and bisect each angle of every triangle with 90% accuracy.

-214
Construct a congruent triangle and bisect the angles.



5.212-215
Given five ordered pairs of integers associated with points on the Cartesian coordinate system, the student is able to list three ordered pairs for each given point that are symmetric to the given point about the X-axis, Y-axis, and Origin with 90% accuracy.

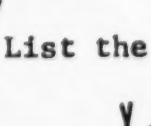
-215
List the ordered pairs that are symmetric to the given point about the X-axis, Y-axis, and Origin.

(+3,-5)

*X-axis (+3,+5)
Y-axis (-3,-5)
Origin (-3,+5)

5.213-216
Given five pairs of similar triangles with whole number measures of four sides of each pair, THE Student is able to list the whole number measures of the other two sides of each pair with 90% accuracy.

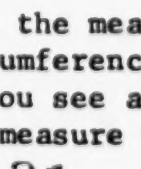
-216
List the measures of the two sides whose measure is unknown.



x : 2
y : 3

5.215-217
Given five right triangles with whole number measures of two sides of each, the student is able to list the measures of the third side of each triangle using the Pythagorean relation with 80% accuracy.

-217
List the measure of the third side.



$3^2 + 4^2 = y^2$

y : 5

5.223-218
Given five circles with identified centers, the student is able to list the measure of the diameter and circumference of each circle and state the relationship that exists between the diameter and circumference of these circles with 90% accuracy.

-218
List the measure of the diameter and circumference of the given circle. Do you see any relationship between the measure of the D and C?

D: 1.2
C: 3.8

es, a compass, and a straightedge, the
to construct angles congruent to the'
nd the angle bisectors of each with

gles, a compass, and a straightedge, the
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and bisect each angle of every triangle
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s of similar triangles with whole number
r sides of each pair, THE Student is
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ch pair with 90% accuracy.

t triangles with whole number measures
each, the student is able to list the
third side of each triangle using the
lation with 80% accuracy.

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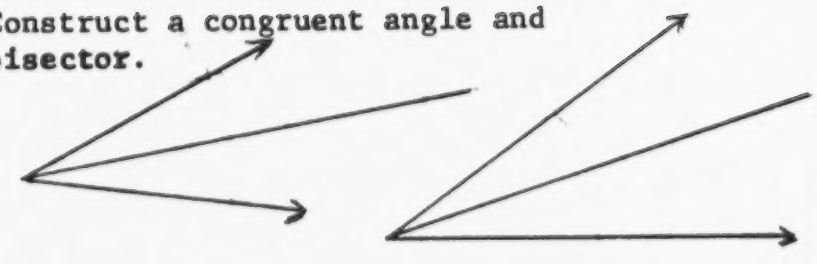
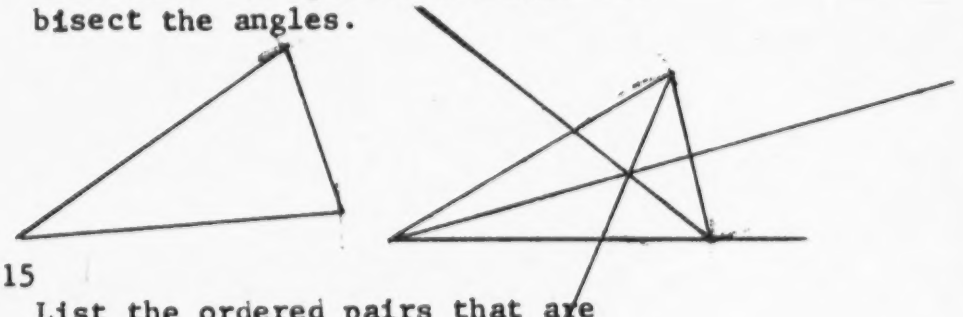
the measure of the diameter and
of each circle and state the relation-
s between the diameter and circumference
s with 90% accuracy.

-213
Construct a congruent angle and bisector.



-214
Construct a congruent triangle and bisect the angles.

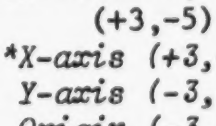


-215
List the ordered pairs that are symmetric to the given point about the X-axis, Y-axis, and Origin.

(+3,-5)

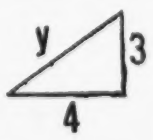
*X-axis (+3,+5)
Y-axis (-3,-5)
Origin (-3,+5)

-216
List the measures of the two sides whose measure is unknown.



x : 2
y : 3

-217
List the measure of the third side.



$3^2 + 4^2 = y^2$

y : 5

-218
List the measure of the diameter and circumference of the given circle. Do you see any relationship between the measure of the D and C?

D : 1.2 cm.
C : 3.8 cm.

5.224-219
Given five parallelograms, the student is able to list the measure of the area of each with 90% accuracy.

5.225-220
Given five models of rectangular prisms, the student is able to list the measure of the volume of each with 80% accuracy.

5.42-221
Given a $1.00 bill, a quarter, a dime, or a nickel, the student is able to make any change, just as a coin changing machine, requested of a fellow student with 100% accuracy.

5.43-222
Given pictures of ten clock faces indicating time on the hour, half hour, or quarter hour, the student is able to list the time on each clock with 90% accuracy.

5.44-223
Given a Fahrenheit thermometer and ten containers with liquids of varying temperature, the student is able to list the temperature of each liquid to the nearest degree with 90% accuracy.

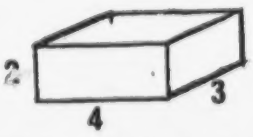
5.45-224
Given ten pairs of measures consisting of the sum of related metric or British units, the student is able to list the indicated sum, difference, product, or quotient with 90% accuracy.

5.45-225
Given twenty pairs of measures such that each pair consists of weight measures(British or metric), the student is able to list the sum of each pair with renaming and 90% accuracy.

-219
List the measure of the area of this parallelogram.

area – 2 sq. cm.

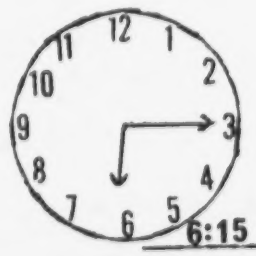
-220
List the measure of the volume of this prism.



vol. – 24 cu. units

-221
Please give me change for this quarter so that I will have 2 nickels.

-222
What time is it?



6:15

-223
Using your thermometer, what is the temperature of container A?

-224
List the indicated sum, or difference.

7 dm. 5cm.	2yds. 2ft. 4 in.
+8 dm. 9cm.	– 1ft. 9 in.
*1m. 6 dm. 4cm.	2 yds. 0ft. 7 in.

-225
List the sum of each pair.
12 lbs. 14 oz. + 25 lbs. 7 oz.
*The sum is 38lbs. 5 oz.

5.46-226
Given a scale and ten objects of varying weights, the student is able to list the measure of the weight of each object to the nearest ounce with 90% accuracy.

5.46-227
Given containers that measure 1 cup, 1 pint, 1 quart, and 1 gallon, the student is able to demonstrate the numerical relationships that exist among these measures with 100% accuracy.

5.214-228
Given a map and a simple scale, the student is able to list the actual distance between ten different pairs of locations within 5 miles and 90% accuracy.

5.22-229
Given ten open mathematical sentences involving the comparison of related metric or British units of measure, the student is able to list the missing member of each sentence with 90% accuracy.

5.221-230
Given ten segments, the student is able to construct a congruent segment of each and list the measure of each to the nearest 1/8 in. or 2/5 cm. with 90% accuracy.

5.221-231
Given an inch and centimeter ruler, the student is able to construct ten segments to the nearest specified 1/4" or ½ cm. with 90% accuracy.

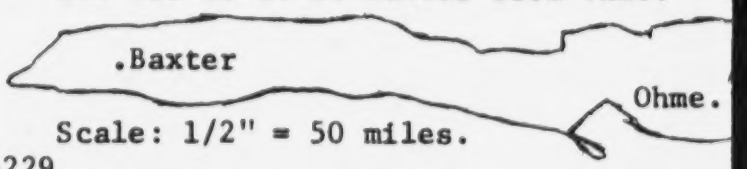
5.221-232
Given a yardstick, the student is able to measure the length and width of his instructiona area in yards, feet, and inches and note the relationship that exists among yards, feet, and inches with 90% accuracy.

5.221-233
Given an inch and centimeter ruler, and ten segments of varying lengths, the student is able to list the measure of each segment to the nearest inch or centimeter with 90% accuracy.

-226
(The student will weigh various obje using a laboratory spring scale.)

-227
Demonstrate the numerical relationship that exists between the cup and the quart.

-228
How far is it to Baxter from Ohme?



Scale: 1/2" = 50 miles.

-229
List the missing member to make each sentence true.

12 lbs. = **192* oz.

-230
Construct a congruent segment and list the measure of the given segment.

A B

m(AB)= 5 4/5 cm.

-231
Construct a segment 2 3/4" long.

-232
Measure your instructional area- length and width= in yards, feet, and inches.What is the relationship among yards, feet, and inches?

-233
List the measure of this segment to the nearest centimeter.

6 cm

le and ten objects of varying weights, the able to list the measure of the weight of to the nearest ounce with 90% accuracy.

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(The student will weigh various objects using a laboratory spring scale.)

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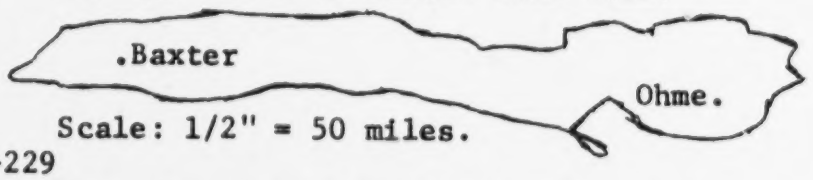
-228
How far is it to Baxter from Ohme?



Scale: 1/2" = 50 miles.

-229
List the missing member to make each sentence true.

12 lbs. = *192 oz.

-230
Construct a congruent segment and list the measure of the given segment.

A B

m(AB)= 5 4/5 cm.

-231
Construct a segment 2 3/4" long.

-232
Measure your instructional area- length and width= in yards, feet, and inches. What is the relationship among yards, feet, and inches?

-233
List the measure of this segment to the nearest centimeter.

6 cm.

5.221-234
Given a unit of linear measure and ten objects of varying lengths, the student is able to list the measure of the length of each object to the nearest unit with 90% accuracy.

5.222-235
The student is able to show, without a protractor, the sum of the measures of the angles of a given triangle.

5.222-236
Given ten angles and a protractor, the student is able to list the measure of each angle to the nearest ½ degree with 90% accuracy.

5.222-237
Given a unit angle whose measure is unknown and ten other angles, the student is able to construct a protractor based on the given unit angle, and list the measure of the ten angles using the constructed protractor with 90% accuracy.

5.223-238
Given ten polygons with the measures of the sides indicated, the student is able to list the measure of the perimeter of each with 90% accuracy.

5.223-239
Given ten squares, triangles, or rectangles, the student is able to list the measure of the perimeter of each using a ruler and with 90% accuracy.

5.224-240
Given five triangles, the student is able to list the measures of the area of each with 90% accuracy.

-234
What is the measure of this
the nearest of our unit of m
3
(unit)
(objec

-235
What is the sum of the measu
triangle?
180°

-236
List the measure of the angl
the nearest ½ degree.
31°

-237
What is the measure of the g
in terms of the unit angle?
(Unit angle)
2 uni

-238
What is the perimeter of thi
1
2
2
10 cm.
1
4

-239
What is the measure of the p
this rectangle using a cm. r
5 cm.

-240
What is the measure of the a
this triangle?

linear measure and ten objects of
, the student is able to list the
length of each object to the nearest
accuracy.

able to show, without a protractor, the
ures of the angles of a given triangle.

es and a protractor, the student is able
asure of each angle to the nearest ½ degree
acy.

ngle whose measure is unknown and ten
he student is able to construct a
ed on the given unit angle, and list the
ten angles using the constructed
n 90% accuracy.

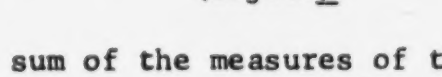
gons with the measures of the sides
student is able to list the measure
er of each with 90% accuracy.

res, triangles, or rectangles, the student
t the measure of the perimeter of each
and with 90% accuracy.

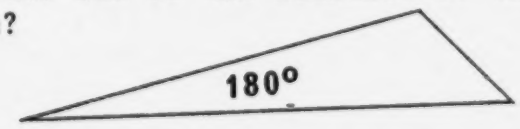
angles, the student is able to list the
e area of each with 90% accuracy.

-234
What is the measure of this object to the nearest of our unit of measure?

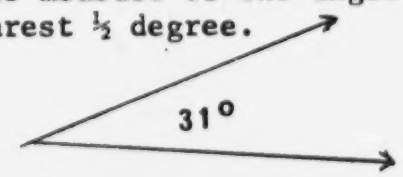
3 (unit)



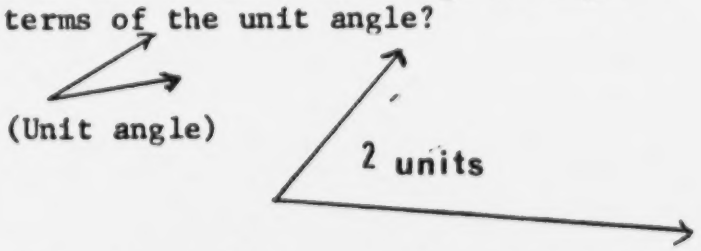
-235
What is the sum of the measures of this triangle?



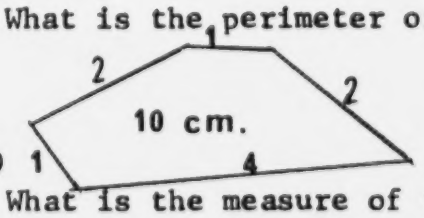
-236
List the measure of the angle to the nearest ½ degree.



-237
What is the measure of the given angle in terms of the unit angle?



-238
What is the perimeter of this pentagon?



-239
What is the measure of the perimeter of this rectangle using a cm. ruler?

-240
What is the measure of the area of this triangle?

5.224-241
Given ten pairs of related measures (feet/inches, etc.) that represent the lengths and widths of rectangles, the student is able to list the measure of the area of the associated rectangular region for each with 90% accuracy.

5.224-242
Given ten rectangular regions with the measures of the sides indicated, the student is able to list the measure of the area of each region with 90% accuracy.

5.225-243
Given five rectangular prisms and an inch or centimeter ruler, the student is able to list the measure of the volume of each prism to the nearest cubic inch or cubic centimeter with 90% accuracy.

5.225-244
Given a cubic-inch container and five three-dimensional objects, the student is able to list the measure of the volume of each figure to the nearest ½ cubic-inch with 100% accuracy.

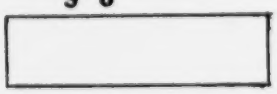
5.226-245
Given ten segments, the student is able to list the measure of each to the nearest 1/2 inch, to the nearest 1/4 inch, to the nearest 1/8 inch, and make a statement regarding possible error in measurement with 90% accuracy.

5.227-246
Given ten measures in inches, feet, or yards, the student is able to list the indicated equivalent measure in yards, feet, or inches with 90% accuracy.

-241
List the measure of the area of each rectangle whose dimensions are:

Length	Width	Area
5ft.6in.	39in.	*2574 sq.in.*

-242
List the measure of each rectangle.



12 8/9 ft.^{2}

-243
List the measure of the given prism in cubic centimeters.

-244
(This one must be done with physical objects.)

-245
What is the measure of this seg ment to the nearest 1/8,1/4, and 1/2 inches? What about possible error in measurement?

2 5/8" — 2 3/4" — 2 1/2"

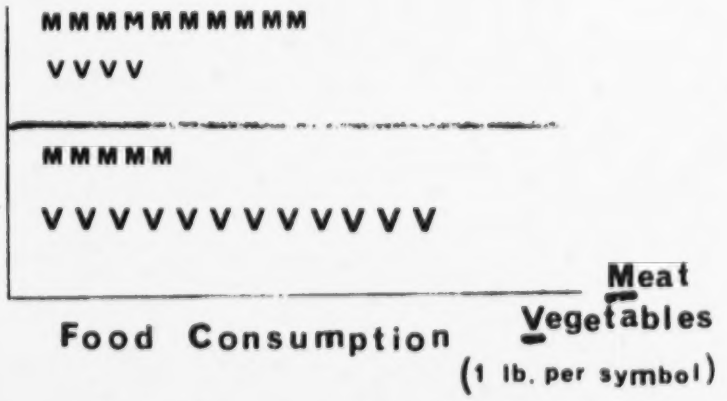
-246
What number makes this open sentence a true statement?
34 inches = *0 yds. 2 ft. 10 in.

6.12-247
Given a Cartesian coordinate system, and a set of 10 ordered pairs of integers, the student is able to associate each ordered pair with a point on the coordinate system with 90% accuracy.

6.13-248
Given a finite lattice ($-8 < x < +8$) and ($-8 < y < +8$), and two sets of points identified by mathematical inequalities, the student is able to graph the intersection of the given sets with 100% accuracy.

6.22-249
Given a set of unorganized data, the student is able to organize the data, construct an appropriate graph of the data, and write five multiple-choice test questions that require interpretation of the data with 100% accuracy.

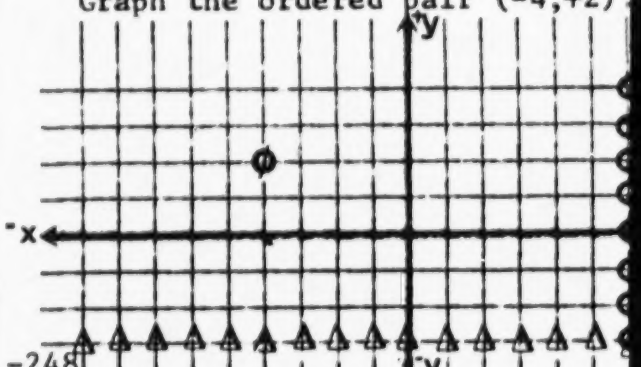
-251



6.221-250
Given a set of unorganized data, the student is able to graph the data, after organizing it, using a bar graph with 90% accuracy.

6.221-251
Given a set of unorganized data, the student is able to construct a pictograph that represents the data.

-247
Graph the ordered pair (-4,+2)

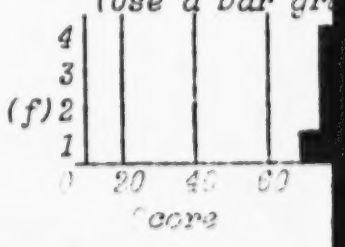


-248
Graph the intersection of (x >
(y<-2)Δ
(Note the correct response ab
247)

-249
Organize the data and use a ci
bar, or pictograph to represen
data. Construct five multiple
questions about the data.

Mary's scores on 20 spelling te
98,99,95,93,96,90,82,75,88,89,
76,98,90,76,87,98,76,72,

* 99 - 1
98 - 3
97- 0
...



-250
(Similar to the objective abov
here we are concerned only wi
graphs.)

-251
Construct a pictograph to repr
data.
During last week, Dan ate 10 l
meat, 4 lbs. of vegetables, Su
lbs. of meet, and 12 lbs. of v
(see left-hand column)

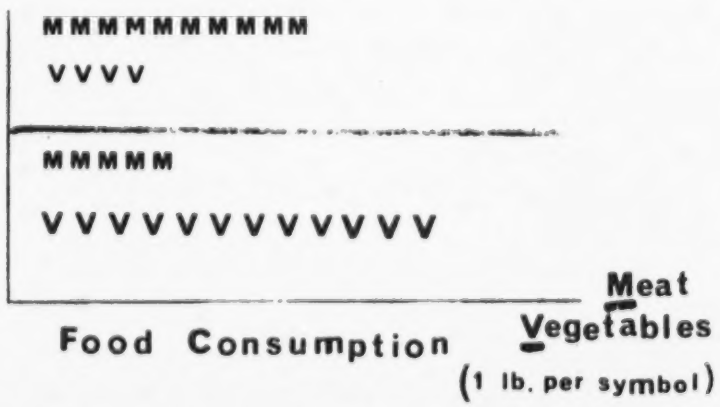
-247
a Cartesian coordinate system, and a set of 10
ed pairs of integers, the student is able to
iate each ordered pair with a point on the
inate system with 90% accuracy.

-248
a finite lattice $(-8 < x < +8)$ and $(-8 < y < +8)$,
two sets of points identified by mathematical
alities, the student is able to graph the
section of the given sets with 100% accuracy.

-249
a set of unorganized data, the student is able
ganize the data, construct an appropriate graph
e data, and write five multiple-choice test
ions that require interpretation of the data
100% accuracy.

an

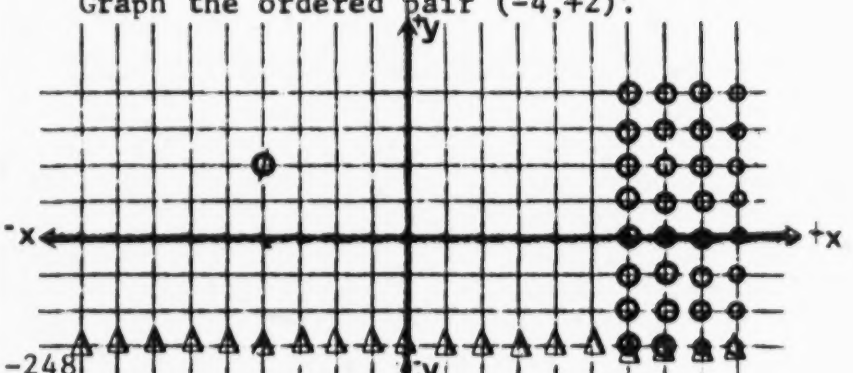
ue



-250
a set of unorganized data, the student is able to
the data, after organizing it, using a bar graph
90% accuracy.

-251
a set of unorganized data, the student is able to
truct a pictograph that represents the data.

-247
Graph the ordered pair (-4,+2)ϕ.

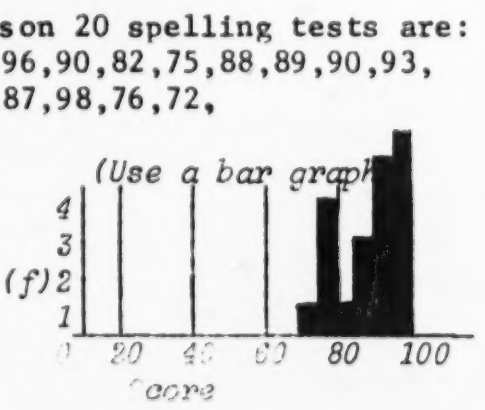


-248
Graph the intersection of $(x > +5)$o and $(y < -2)\Delta$
(Note the correct response above in 247)

-249
Organize the data and use a circle, bar, or pictograph to represent the data. Construct five multiple-choice questions about the data.

Mary's scores on 20 spelling tests are:
98,99,95,93,96,90,82,75,88,89,90,93,
76,98,90,76,87,98,76,72,

* 99 - 1
98 - 3
97- 0
...



-250
(Similar to the objective above except here we are concerned only with bar graphs.)

-251
Construct a pictograph to represent the data.
During last week, Dan ate 10 lbs. of meat, 4 lbs. of vegetables, Sue ate 5 lbs. of meet, and 12 lbs. of vegetables.
(see left-hand column)

6.23-252
Given an unorganized set of data, the student is able to organize the data and list the mean, median, and mode with 100% accuracy.

-252
List the mean, median, and mode for this set of data.

1,1,3,3,7,7,7,5,5,9,9,9,9
Mode - 9, Median- 7, Mean - 5.8

6.231-253
Given ten sets of numerals that name whole numbers less than 30, the student is able to list the mean (no remainders in division) of each set with 90% accuracy.

-253
List the mean.
1,2,3,4,5,6,7
Mean-4

7.2-254
Given two regular tetrahedra of different colors and on each of whose faces is a different symbol, the student is able to list the sample space for rolling the tetrahedra, list the probability associated with each member of the sample space, and predict the occurence of each member of the sample space in 100 tosses of the tetrahedra after which he completes the experiment to match his predictions with a single experiment.

-254
The sample space for rolling two tetrahedra would be-
*{(1,A), (1,B),(1,C),(2,A),...,(3,B),(3,C)}
The probability associated with each member of the sample space-
*1/9

In 100 tosses of the tetrahedra I would expect each member of the sample space to occur-

*about 11.
(the student would then complete the experiment noting that the theoretical probability and experimental results are often different.)

ELEMENTARY MATHEMATICS CONTENT GUIDE

This Guide represents an attempt to outline the content of elementary school mathematics that would provide the necessary range for a program of individualized mathematics instruction consistent with the philosophy of the Rationale For Individualized Instruction. Naturally, all elementary students will not, and should not, be expected to master concepts and content associated with each portion of this Guide. As a corallary to that statement it should be recognized that for certain students this Guide may not be completely adequate.

All too often content outlines consist of words that fail to communicate to the reader. To help alleviate this problem and to tie the content portion of the Guide to materials that might be found in the elementary classroom, mathematical explanations and sample problems are listed.

You will note that the decimal system of subdivision has been used in the development of this Guide. It is a convenient system which allows the author to relate a number of items in this volume. The reader will do himself a favor if he adopts this outline system for his own personal use.

References used in the development of this volume include:

SMSG, *Elementary School Mathematics*, all volumes K-6; Nichols, Eugene, *Elementary Mathematics: Patterns and Structure*, all volumes K-8; Pennsylvania Department of Public Instruction, Emmanuel Berger, Research Associate, *Primes: Mathematics Content Authority List K-6, (abridged)*; Koch, Dale, *Content Outline for Mathematics, Chester Park Individualized Project*; and Ohmer, Merlin, *Elementary Contemporary Mathematics*; Waltham, Massachusetts Blaisdell Publishing Co., 1964.

ELEMENTARY MATHEMATICS CONTENT GUIDE

Content	Mathematical Explanation	Sample Pr
1. Logical Reasoning		
1.1 Simple Sentences and their truth values (Open equalities and inequalities-math or Eng.)	1.1 Symbolized as p,q,r, etc.	1.1 p: q:
1.2 Compound Sentences and their truth values		
1.21 Negation	1.21 Symbolized $\sim p$ (read "not-p")	1.21 p ~p
1.22 Disjunction	1.22 Symbolized $p \vee q$ (read "p or q")	1.22 p q p
1.23 Conjunction	1.23 Symbolized $p \wedge q$ (read "p and q")	1.23 p q p
1.24 Conditional	1.24 Symbolized $p \rightarrow q$ (read "if p, then q")	1.24 p q p
1.25 Biconditional	1.25 Symbolized $p \leftrightarrow q$ (read "p if and only if q")	1.25 p q p
1.3 Inference schemes		
*** *** *** *** *** ***	*** *** ***	***
2. Sets (set is undefined)		
2.1 Concepts		
2.11 Subset	2.11 $^{*}A \subset B \leftrightarrow (x \in A \rightarrow x \in B)$	2.11 l
2.12 Equal sets	2.12 $A=B \leftrightarrow (A \subset B \wedge B \subset A)$	2.12 a
2.13 Equivalent sets	2.13 $^{*}A \approx B \leftrightarrow$ each element of A corresponds to one element of B, and each element of B corresponds exactly to one element of A.	2.13
2.14 Finite sets	2.14 Any set A is finite if and only if it is not infinite.	2.14 u

$^{*}\subset$ "is a subset of"

$^{*}\in$ "is a member of"

$^{*}\approx$ "is equivalent to"

"is in one-to-one correspondence with"

ELEMENTARY MATHEMATICS CONTENT GUIDE

ntent	Mathematical Explanation	Sample Problem
easoning		
e *Sentences and their truth values* (equalities and inequalities-math or Eng.)	1.1 *Symbolized as* p,q,r, etc.	1.1 p: 2 + 3 ≠ 5 (false) q: 3 x 0 = 0 (true)
und Sentences and their truth values		
Negation	1.21 Symbolized ~p (read "not-p")	1.21 p: 2 + 3 ≠ 5 ~p is true.
Disjunction	1.22 Symbolized p ∨ q (read "p or q")	1.22 p: 2 + 3 ≠ 5 q: 3 x 0 = 0 p ∨ q is true.
Conjunction	1.23 Symbolized p ∧ q (read "p and q")	1.23 p: 2 + 3 ≠ 5 q: 3 x 0 = 0 p ∧ q is false.
Conditional	1.24 Symbolized p ⟶ q (read "if p, then q")	1.24 p: 2 + 3 ≠ 5 q: 3 x 0 = 0 p ⟶ q is true.
Biconditional	1.25 Symbolized p ⟷ q (read "p if and only if q")	1.25 p: 2 + 3 ≠ 5 q: 3 x 0 = 0 p ⟷ q is false.
ence schemes		
*** *** *** ***	*** *** ***	*** *** *** ***
t is undefined)		
pts		
Subset	2.11 $*A \subset B \leftrightarrow (x \in A \rightarrow x \in B)$	2.11 $\{1,2\}$ is a subset of $\{3,2,1\}$
Equal sets	2.12 $A=B \leftrightarrow (A \subset B \wedge B \subset A)$	2.12 $\{a,b\} = \{b,a\}$
Equivalent sets	2.13 $*A \approx B \leftrightarrow$ each element of A corresponds to one element of B, and each element of B corresponds exactly to one element of A.	2.13 $\{m,n,o\} \approx \{\dot{z},x,c\}$
Finite sets	2.14 Any set A is finite if and only if it is not infinite.	2.14 $\{\upsilon,\varphi,\chi,\psi,\omega,\vartheta,\phi,\varsigma,\sigma\}$ is finite

$*\subset$ "is a subset of"

$*\in$ "is a member of"

$*\approx$ "is equivalent to"

"is in one-to-one correspondence with"

2.15 Infinite Sets	2.15 Any set A is infinite $\leftrightarrow$ there is a non-empty proper subset of A which is equivalent to A.	2.15 A = {0,1,2,3,4,.. since B = {1,3,5
2.16 Universal sets	2.16 The set U of all elements under discussion.	2.16 U = {0,1,2,
2.17 Empty set	2.17 Symbolized $\emptyset$ or $\{\ \}$	2.17 A = {x: x i and x A = $\emptyset$
2.18 Disjoint sets	2.18 $A \cap B = \emptyset$	2.18 C = {2,4,6,8 D = {1,3,5,7 C $\cap$ D = $\emptyset$; t d
2.2 Set Operations		
2.21 Union	2.21 $A \underline{\cup} B = \{x : x \in A \vee x \in B\}$ *	2.21 A = {1,2,3
2.22 Intersection	2.22 $A \cap B = \{x : x \in A \wedge x \in B\}$	2.22 A = {1,2,3} A $\underline{\cup}$ B = A $\cap$ B =
2.23 Cartesian product (cross-product)	2.23 $A \times B = \{(a,b) : a \in A \wedge b \in B\}$	2.23 E = {c,d,e,f E X G = {(c,1), (d,2), (f,1),
2.24 Complementation	2.24 $\bar{B} = \{x : x \in U \wedge x \notin B\}$	2.24 U is the set B = {0, +1, $\bar{B}$ = {...,-3,
2.3 Properties of Operations on Sets		
2.31 Union		
2.311 Closure	2.311 $\forall A,B$ $A \underline{\cup} B$ is a unique set.	2.311 A = {0,1,2 A $\underline{\cup}$ B = {0,

* $\underline{\cup}$ symbolizes union to distinguish the use of U as the symbol for Universal set.

nfinite Sets	2.15 Any set A is infinite ⟷ there is a non-empty proper subset of A which is equivalent to A.	2.15 A = $\{0,1,2,3,4,\ldots\}$ is infinite, since B = $\{1,3,5,7,\ldots\}$, A ≈ B, A ⊂ A
niversal sets	2.16 The set U of all elements under discussion.	2.16 U = $\{0,1,2,3,4,\ldots\}$
mpty set	2.17 Symbolized $\emptyset$ or $\{\ \}$	2.17 A = $\{x: x$ is a human being and x is 20 ft. tall$\}$ A = $\emptyset$
isjoint sets	2.18 $A \cap B = \emptyset$	2.18 C = $\{2,4,6,8,\ldots\}$ D = $\{1,3,5,7,\ldots\}$ $C \cap D = \emptyset$; the sets are disjoint
erations nion	2.21 $A \underline{\cup} B = \{x: x \in A \vee x \in B\}$ *	2.21 A = $\{1,2,3\}$ B = $\{0,2,4\}$ $A \underline{\cup} B = \{0,1,2,3,4\}$
ntersection	2.22 $A \cap B = \{x: x \in A \wedge x \in B\}$	2.22 A = $\{1,2,3\}$ B = $\{0,2,4\}$ $A \cap B = \{2\}$
artesian product (cross-product)	2.23 $A \times B = \{(a,b): a \in A \wedge b \in B\}$	2.23 E = $\{c,d,e,f\}$ G = $\{1,2\}$ E X G = $\{(c,1), (c,2), (d,1), (d,2), (e,1), (e,2), (f,1), (f,2)\}$
omplementation	2.24 $\tilde{B} = \{x: x \in U \wedge x \notin B\}$	2.24 U is the set of integers: B = $\{0, +1, +2, +3, \ldots\}$ $\tilde{B}$ = $\{\ldots,-3,-2,-1\}$
ties of Operations on Sets nion .311 Closure	2.311 $\forall A,B$ $A \underline{\cup} B$ is a unique set.	2.311 A = $\{0,1,2\}$ B = $\{z,x\}$ $A \underline{\cup} B$ = $\{0,1,2,z,x\}$ which itself is a set.

* $\underline{\cup}$ symbolizes union to distinguish the use of U as the symbol for Universal set.

2.312 Commutative	2.312 $\forall^{*}_{A,B}\left(A \underline{U} B = B \underline{U} A\right)$	2.312 $A = \{\Omega, \Delta$
2.313 Associative	2.313 $\forall_{A,B,C}\left[\begin{matrix}(A \underline{U} B) \underline{U} C = \\ A \underline{U} (B \underline{U} C)\end{matrix}\right]$	$A \underline{U} B = \{\Omega,$ $B \underline{U} A = \{\phi,$ 2.313 (Use se $C = \{\eta$ $(A \underline{U} B) \underline{U} C$ $A \underline{U}(B \underline{U} C) = \Omega$
2.314 Identity	2.314 $\forall_{A}\left(A \underline{U} \emptyset = A\right)$	2.314 using se $A \underline{U} \emptyset = A$
2.315 Complement	2.315 $\forall_{A}\left(A \underline{U} \tilde{A} = U\right)$	2.315 U = the se A = the se $\tilde{A}$ = the se $A \underline{U} \tilde{A} = U$
2.316 Idempotent	2.316 $\forall_{A}\left(A \underline{U} A = A\right)$	2.316 A = the se $A \underline{U} A = A$
2.32 Intersection		
2.321 Closure	2.321 $\forall_{A,B}(A \cap B)$ is a unique set.	2.321 A = the se B = the se $A \cap B = \emptyset$ which
2.322 Commutative	2.322 $\forall_{A,B}\left(A \cap B = B \cap A\right)$	2.322 $A = \{X$ $B = \{\Psi$ $A \cap B = \{\Psi$ $B \cap A = \{\Psi$
2.323 Associative	2.323 $\forall_{A,B,C}\left[\begin{matrix}(A \cap B) \cap C = \\ A \cap (B \cap C)\end{matrix}\right]$	2.323 (Use sets $C = \{6\}$ $(A \cap B) \cap C =$ $A \cap (B \cap C) =$
2.324 Identity	2.324 $\forall_{A}(A \cap U) = A$	2.324 U= the set A= the set $A \cap U = A$
2.325 Complement	2.325 $\forall_{A}(A \cap \tilde{A}) = \emptyset$	2.325 A= the set $\tilde{A}$= the set $A \cap \tilde{A} = \emptyset$

* $\forall$ symbolizes the phrase "for all" or "for every"

mmutative	2.312 $\forall^{*}_{A,B}\left(A \underline{U} B = B \underline{U} A\right)$	2.312 $A = \{\Omega, \Delta, \omega, \partial\}$ $B = \{\phi, \theta\}$ $A \underline{U} B = \{\Omega, \Delta, \omega, \partial, \phi, \theta\}$ $B \underline{U} A = \{\phi, \theta, \Omega, \Delta, \omega, \partial\}$
sociative	2.313 $\forall_{A,B,C}\left[\begin{matrix}(A \underline{U} B) \underline{U} C = \\ A \underline{U} (B \underline{U} C)\end{matrix}\right]$	2.313 (Use sets A and B of 2.312) $\wedge$ $C = \{\eta\}$ $(A \underline{U} B) \underline{U} C = \{(\Omega, \Delta, \omega, \partial, \phi, \theta)\eta\}$ $A \underline{U} (B \underline{U} C) = \{\Omega, \Delta, \omega, \partial, (\phi, \theta, \eta)\}$
ntity	2.314 $\forall_{A}\left(A \underline{U} \emptyset = A\right)$	2.314 using set A of 2.312, $A \underline{U} \emptyset = A$
plement	2.315 $\forall_{A}\left(A \underline{U} \tilde{A} = U\right)$	2.315 U = the set of whole numbers A = the set of even numbers $\tilde{A}$ = the set of odd numbers $A \underline{U} \tilde{A} = U$
mpotent	2.316 $\forall_{A}\left(A \underline{U} A = A\right)$	2.316 A = the set of whole numbers $A \underline{U} A = A$
ion sure	2.321 $\forall_{A,B}\left(A \cap B\right)$ is a unique set.	2.321 A = the set of odd numbers B = the set of even numbers $A \cap B = \emptyset$ which itself is a set.
mutative	2.322 $\forall_{A,B}\left(A \cap B = B \cap A\right)$	2.322 $A = \{X, \Psi, \Pi\}$ $B = \{\Psi, \Upsilon, \Pi\}$ $A \cap B = \{\Psi, \Pi\}$ $B \cap A = \{\Psi, \Pi\}$
sociative	2.323 $\forall_{A,B,C}\left[\begin{matrix}(A \cap B) \cap C = \\ A \cap (B \cap C)\end{matrix}\right]$	2.323 (Use sets A and B of 2.322) $\wedge$ $C = \{\sigma\}$ $(A \cap B) \cap C = \phi$ $A \cap (B \cap C) = \phi$
entity	2.324 $\forall_{A}\left(A \cap U\right) = A$	2.324 U= the set of whole numbers A= the set of multiples of 3 $A \cap U = A$
mplement	2.325 $\forall_{A}\left(A \cap \tilde{A}\right) = \emptyset$	2.325 A= the set of odd numbers $\tilde{A}$= the set of even numbers $A \cap \tilde{A} = \emptyset$

* $\forall$ symbolizes the phrase "for all" or "for every"

2.326 Idempotent	$2.326\ \forall A\ (A \cap A) = A$	2.326 A = the set of who $A \cap A = A$
2.33 Combined Operations 2.331 Intersection over Union	$2.331\ \forall A,B,C\ [A \cap (B \cup C) = (A \cap B) \cup (A \cap C)]$	2.331 A = multiples of 3 B = prime numbers C = even numbers 1 $A \cap (B \cup C)$ = the set are 3 an $(A \cap B) \cup (A \cap C)$ = the elements a
2.332 Union over Intersection	$2.332\ \forall A,B,C\ [A \cup (B \cap C) = (A \cup B) \cap (A \cup C)]$	2.332 Using the sets give we have: $A \cup (B \cap C)$ = the se are 2,3,6,9. $(A \cup B) \cap (A \cup C)$ = elements are 2,3
2.4 Equivalence relations 2.41 Definition on any set S 2.42 Properties of equivalence relations 2.421 Reflexive	$2.421\ \forall S\ (a \in S \rightarrow a\ R\ a)$	2.41 Many relations could a given set S; howev would not all be equ 2.421 relations. Consider "is of the same colo If we have a set of would a single rod b same color as itself
2.422 Symmetric	$2.422\ \forall S\ (a,b \in S \wedge a\ R\ b \rightarrow b\ R\ a)$	2.422 If a rod is the same second rod, would th be the same color as
2.423 Transitive	$2.423\ \forall S(a,b,c \in S \wedge a\ R\ b \wedge b\ R\ c \rightarrow a\ R\ c)$	2.423 If a rod a is the sa rod b, and rod b is as rod c, would rod color as rod c? (I h

"is the same color as" is an equivalence relation.

"is the father of"?

empotent

$2.326\ \forall A\ (A \cap A) = A$

2.326 A = the set of whole numbers
$A \cap A = A$

Operations
tersection over Union

$2.331\ \forall A,B,C\ [A \cap (B \cup C) = (A \cap B) \cup (A \cap C)]$

2.331 A = multiples of 3 less than 10
B = prime numbers less than 10
C = even numbers less than 10
$A \cap (B \cup C)$ = the set whose elements are 3 and 6
$(A \cap B) \cup (A \cap C)$ = the set whose elements are 3 and 6

ion over Intersection

$2.332\ \forall A,B,C\ [A \cup (B \cap C) = (A \cup B) \cap (A \cup C)]$

2.332 Using the sets given in 2.331, we have:
$A \cup (B \cap C)$ = the set whose element are 2,3,6,9.
$(A \cup B) \cap (A \cup C)$ = the set whose elements are 2,3,6,9

elations
on on any set S
es of equivalence
s

2.41 Many relations could be defined on a given set S; however, these would not all be equivalence relations. Consider the relation "is of the same color as".

flexive

$2.421\ \forall S\ (a \in S \rightarrow a\ R\ a)$

2.421 If we have a set of Cuisenaire rods would a single rod be the be the same color as itself? (yes)

mmetric

$2.422\ \forall S\ (a,b \in S \wedge a\ R\ b \rightarrow b\ R\ a)$

2.422 If a rod is the same color as a second rod, would the second rod be the same color as the first?yes

ansitive

$2.423\ \forall S(a,b,c \in S \wedge a\ R\ b \wedge b\ R\ c \rightarrow a\ R\ c)$

2.423 If a rod a is the same color as a rod b, and rod b is the same color as rod c, would rod a be the same color as rod c? (I hope so.)

"is the same color as" is an equivalence relation. What about "is the father of"?

3. Number Systems
3.1 Whole Number System
3.11 Definition of a Whole Number

	3.11 Let S be the set of all sets. Let the relation R on S be " is in 1-to-1 correspondence with." R is an equivalence relation on S, thus partitioning S into mutually disjoint subsets. Any one of these subsets is defined to be a whole number.	3.11 One of … would b… the num… the wor… This su… picture… { … { … { …
3.12 Operations		
3.121 Addition, a binary operation		
3.1211 Definition	3.1211 $\forall a,b \in W\ (a + b) =$ $n(A \underline{\cup} B)$, where $n(A)=a$, $n(B)=b$, and $A \cap B = \emptyset$.	3.1211 A = { … B = { … 3 + 2 = 5 = … A ∩ B = …
3.1212 Properties		
3.12121 Closure	3.12121 $\forall a,b \in W\ (a + b) \in W$	3.12121 (9 + 8…
3.12122 Commutative	3.12122 $\forall a,b \in W\ a + b = b + a$	3.12122 (14 + …
3.12123 Associative	3.12123 $\forall a,b,c \in W\ (a + b) + c = a + (b + c)$	3.12123 (7 + 4… 11 …
3.12124 Identity	3.12124 $\forall a \in W\ (a + 0) = a$	3.12124 9 + 0…
3.1213 Computation and Word Problems		
3.12131 Two Addends - Basic facts - Numbers greater than 9 without renaming - Numbers greater than 9 with renaming	3.12131	3.12131 3 + 9… 132 + … 2436 + …

s ber System nition of a Whole Number	3.11 Let S be the set of all sets. Let the relation R on S be " is in 1-to-1 correspondence with." R is an equivalence relation on S, thus partitioning S into mutually disjoint subsets. Any one of these subsets is defined to be a whole number.	3.11 One of the disjoint subsets would be associated with the numeral name " 3 " or the word name "three". This subset might be pictured as follows: $\{\rho, \rho, \rho\}, \{\sigma, \eta, \mu\}, \{\Delta, \Delta, \Delta\}, \ldots$
ations 1 Addition, a binary operation 3.1211 Definition	3.1211 $\forall a,b \in W\ (a + b) = n(A \cup B)$, where $n(A)=a$, $n(B)=b$, and $A \cap B = \emptyset$.	3.1211 $A = \{\theta, \Xi, \Psi\}$, $n(A) = 3$ $B = \{\gamma, \Delta\}$, $n(B) = 2$ $3 + 2 = 5 = n(A \cup B)$ where $A \cap B = \emptyset$
3.1212 Properties		
3.12121 Closure	3.12121 $\forall a,b \in W\ (a + b) \in W$	3.12121 $(9 + 8) \in W$
3.12122 Commutative	3.12122 $\forall a,b \in W\ a + b = b + a$	3.12122 $(14 + 29) = (29 + 14)$
3.12123 Associative	3.12123 $\forall a,b,c \in W\ (a + b) + c = a + (b + c)$	3.12123 $(7 + 4) + 2 = 7 + (4 + 2)$ $11 + 2 = 7 + 6$ $13 = 13$
3.12124 Identity	3.12124 $\forall a \in W\ (a + 0) = a$	3.12124 $9 + 0 = 9$
3.1213 Computation and Word Problems		
3.12131 Two Addends - Basic facts - Numbers greater than 9 without renaming - Numbers greater than 9 with renaming	3.12131	3.12131 $3 + 9 = 12$ $132 + 225 = 357$ $2436 + 875 = 3311$

3.12132 Three or more addends -Basic facts without renaming -Basic facts with renaming -Numbers greater than 9 without renaming -Numbers greater than 9 with renaming	3.12132	3.12132 3 + 4 + 2 = 9 4 + 7 + 8 + 5 11 + 32 + 25 234 + 469 + 7
3.12133 Machine Calculation		
3.122 Subtraction, a binary operation		
3.1221 Definition	3.1221 $^{*}\forall_{a,b \in W}\ (b - a) =$ $n(B \setminus A)$ where $n(A)=a$, $n(B) = b$, and $A \subset B$.	3.1221 B = {π, x A = {ω, x} b - a = n(B \ A)
3.1222 Computation and Word Problems		
3.12221 Basic facts		3.12221 7 - 2 = 5
3.12222 Numbers greater than 9 without renaming		3.12222 29 - 12 = 17
3.12223 Numbers greater than 9 with renaming		3.12223 312 - 156 =
3.12224 Machine Calculation		
3.123 Multiplication, a binary operation		
3.1231 Definition	3.1231 $\forall_{a,b \in W}$ ab =n(AxB) (An alternate definition which consists of the number of the union of a sets with b in each set is not included here, though quite appropriate.)	3.1231 A = {ω, B = {σ, A x B = {(ω,σ), (Φ,π), n(A x B) = 6 Therefore, 3 x 2 =
3.1232 Properties		
3.12321 Closure	3.12321 $\forall_{a,b \in W}$ (ab) $\in$ W	3.12321 (9 x 7) ∈
3.12322 Commutative	3.12322 $\forall_{a,b \in W}$ (ab)=(ba)	3.12322 (4 x 3)
3.12323 Associative	3.12323 $\forall_{a,b,c \in W}$ (ab)c= a(bc)	3.12323 (3 x 4) x 12 x 5 60 =

$^{*}B \setminus A = \{\alpha : \alpha \in B \wedge \alpha \notin A\}$

3.12132 Three or more addends -Basic facts without renaming -Basic facts with renaming -Numbers greater than 9 without renaming -Numbers greater than 9 with renaming	3.12132	3.12132 3 + 4 + 2 = 9 4 + 7 + 8 + 5 = 24 11 + 32 + 25 = 68 234 + 469 + 7921 = 8624
3.12133 Machine Calculation		
btraction, a binary operation		
1221 Definition	3.1221 $\overset{*}{\forall}_{a,b \in W}\ (b - a) = n(B \setminus A)$ where $n(A)=a$, $n(B) = b$, and $A \subset B$.	3.1221 $B = \{\pi, \varkappa, \omega, \Omega\}$ $n(B)=4$ $A = \{\omega, \varkappa\}$ $n(A)=2$ $b - a = n(B \setminus A) = 2$
1222 Computation and Word Problems		
3.12221 Basic facts		3.12221 $7 - 2 = 5$
3.12222 Numbers greater than 9 without renaming		3.12222 $29 - 12 = 17$
3.12223 Numbers greater than 9 with renaming		3.12223 $312 - 156 = 156$
3.12224 Machine Calculation		
ltiplication, a binary operation		
1231 Definition	3.1231 $\forall_{a,b \in W}\ ab = n(A \times B)$ (An alternate definition which consists of the number of the union of $\underline{a}$ sets with $\underline{b}$ in each set is not included here, though quite appropriate.)	3.1231 $A = \{\omega, \Phi, \eta\}$ $n(A)=3$ $B = \{\sigma, \pi\}$ $n(B)=2$ $A \times B = \{(\omega,\sigma), (\omega,\pi), (\Phi,\sigma), (\Phi,\pi), (\eta,\sigma), (\eta,\pi)\}$ $n(A \times B) = 6$ Therefore, $3 \times 2 = 6$
1232 Properties		
3.12321 Closure	3.12321 $\forall_{a,b \in W}\ (ab) \in W$	3.12321 $(9 \times 7) \in W$
3.12322 Commutative	3.12322 $\forall_{a,b \in W}\ (ab)=(ba)$	3.12322 $(4 \times 3) = (3 \times 4)$
3.12323 Associative	3.12323 $\forall_{a,b,c \in W}\ (ab)c = a(bc)$	3.12323 $(3 \times 4) \times 5 = 3 \times (4 \times 5)$ $12 \times 5 = 3 \times 20$ $60 = 60$

$^{*}\ B \setminus A = \{\alpha : \alpha \in B \wedge \alpha \notin A\}$

3.12324 Zero property	3.12324 $\forall_{a \in W}\ a \times 0 = 0$	3.12
3.12325 Identity	3.12325 $\forall_{a \in W}\ a \times 1 = a$	3.12
3.12326 Distributive	3.12326 $\forall_{a,b,c \in W}\ a(b + c) = ab + ac$	3.12
3.1233 Computation and Word Problems		
3.12331 Two factors -Basic facts -Numbers greater than 10 without renaming -Numbers greater than 10 with renaming	3.12331	3.12
3.12332 Three or more factors without renaming		3.12
3.12333 Three or more factors with renaming		3.12
3.12334 Multiples of ten as a factor		3.12
3.12335 Powers of ten as a factor		3.12
3.12336 Machine calculation		
3.124 Division, a binary operation		
3.1241 Definition	3.1241 $\forall_{a,b,c \in W}\ a \div b = c \longleftrightarrow cb = a$	3.12
3.1242 Computation and Word Problems	3.1242 The Division algorithm for whole numbers is defined: $\forall_{a \neq 0, b \neq 0 \in W},\ b < a,$ $\exists\ q \neq 0, r \neq 0 \in W \ni a = bq + r$ and $0 \leq r < b$. **	
3.12421 Basic facts		3.12
3.12422 Known factor (divisor) less than 10		3.12
3.12423 Known factor greater than 10		3.12
3.12424 Known factor a multiple of 10	3.1243 $\forall_{a,b,c \neq 0 \in W}(a+b) \div c = (a \div c) + (b \div c)$	3.12
3.12425 Known factor a power of 10		3.12
3.12426 Machine calculation		3.12

** $\exists$ symbol represents the phrase "there exists" and $\ni$ symbol represents the phrase "such that".

3.1243 Right distributive property

3.12324 Zero property	3.12324 $\forall a \in W \;\; a \times 0 = 0$	3.12324	$9 \times 0 = 0$
3.12325 Identity	3.12325 $\forall a \in W \;\; a \times 1 = a$	3.12325	$321 \times 1 = 321$
3.12326 Distributive	3.12326 $\forall a,b,c \in W \;\; a(b + c) = ab + ac$	3.12326	$3 \times (4 + 10) = (3 \times 4) + (3 \times 10)$
33 Computation and Word Problems			
3.12331 Two factors -Basic facts -Numbers greater than 10 without renaming -Numbers greater than 10 with renaming	3.12331	3.12331	$4 \times 8 = 32$ $12 \times 24 = 188$ $25 \times 36 = 900$
3.12332 Three or more factors without renaming		3.12332	$2 \times 4 \times 9 = 72$
3.12333 Three or more factors with renaming		3.12333	$12 \times 26 \times 128 = 39936$
3.12334 Multiples of ten as a factor		3.12334	$20 \times 56 = 1120$
3.12335 Powers of ten as a factor		3.12335	$1000 \times 69 = 69000$
3.12336 Machine calculation			
sion, a binary operation			
241 Definition	3.1241 $\forall a,b,c \in W \;\; a \div b = c \leftrightarrow cb = a$	3.1241	$32 \div 4 = 8 \leftrightarrow 8 \times 4 = 32$
242 Computation and Word Problems	3.1242 The Division algorithm for whole numbers is defined: $\forall a \neq 0, b \neq 0 \in W,\ b < a,$ $\exists\ q \neq 0, r \neq 0 \in W \ni a = bq + r$ and $0 \leq r < b$. **		
3.12421 Basic facts		3.12421	$21 \div 7 = 3$
3.12422 Known factor (divisor) less than 10		3.12422	$429 \div 6 = 71$ r. 3
3.12423 Known factor greater than 10		3.12423	$231 \div 15 = 15$ r. 6
3.12424 Known factor a multiple of 10	3.1243 $\forall a,b,c \neq 0 \in W (a + b) \div c = (a \div c) + (b \div c)$	3.12424	$1392 \div 30 = 46$ r. 12
3.12425 Known factor a power of 10		3.12425	$8590 \div 100 = 85$ r. 90
3.12426 Machine calculation			
43 Right distributive property		3.1243	$(28 + 7) \div 7 = (28 \div 7) + (7 \div 7)$

** $\exists$ symbol represents the phrase "there exists" and $\ni$ symbol represents the phrase "such that".

3.13 Order relation "is less than"

3.131 Definition

3.131 $\forall a,b \in W \quad a < b \longleftrightarrow \exists A,B \ni n(A) = a$ and $n(B)=b$ and $A \subseteq B$.

3.132 Properties

3.1321 Transitivity

3.1321 $\forall a,b,c \in W \ (a < b) \wedge (b < c) \longrightarrow (a < c)$.

3.1322 Trichotomy

3.1322 $\forall a,b \in W$ one and only one of the following is true: $a < b$; $a = b$; $b < a$.

3.14 Numeration Systems

3.141 Base-ten numeration (Hindu-Arabic)

3.1411 Expanded Notation

3.1412 Reading and writing names for whole numbers

3.1413 Rounding and Estimation

3.1414 Exponential notation

3.1415 Scientific notation

3.142 Non-decimal place-value systems

3.1421 Bases 2,5,8,12, and 20

3.1422 Operations

3.143 Historical systems of notation

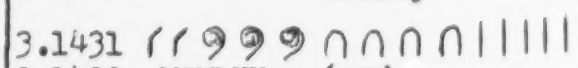
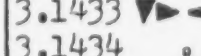
3.1431 Egyptian

(20,345)

3.1432 Roman

3.1433 Babylonian

3.1434 Mayan

relation "is less than" Definition	3.131 $\forall a,b \in W$ $a < b \longleftrightarrow$ $\exists A,B \ni n(A) = a$ and $n(B)=b$ and $A \subseteq B$.	3.131 $A = \{\zeta, \lambda\}$, $n(A) = 2$ $B = \{\mu, \zeta, \lambda\}$, $n(B) = 3$ Since $A \subseteq B$, $2 < 3$.
Properties		
3.1321 Transitivity	3.1321 $\forall a,b,c \in W$ $(a < b) \wedge (b < c) \rightarrow (a < c)$.	3.1321 $(3 < 4) \wedge (4 < 7) \rightarrow (3 < 7)$
3.1322 Trichotomy	3.1322 $\forall a,b \in W$ one and only one of the following is true: $a < b$; $a = b$; $b < a$.	3.1322 $4,5 \in W$; one and only one of the following is true: $4 < 5$; $4 = 5$; $5 < 4$. Based on the definition of "<" $4 < 5$ is true.
ation Systems		
Base-ten numeration (Hindu-Arabic)		
3.1411 Expanded Notation		3.1411 $354 = 300 + 50 + 4$
3.1412 Reading and writing names for whole numbers		3.1412 The word name for 354 is three hundred fifty-four.
3.1413 Rounding and Estimation		3.1413 Round 56 to the nearest multiple of 10- (60) " " 100- (100)
3.1414 Exponential notation		3.1414 $1253 = 1x10^3 + 2x10^2 + 5x10^1 + 3x10^0$
3.1415 Scientific notation		3.1415 $1253 = 1.253 \times 10^3$
Non-decimal place-value systems		
3.1421 Bases 2,5,8,12, and 20		3.1421 $15_{ten} = 30_{five}$
3.1422 Operations		3.1422 $15_{eight} + 2E_{twelve} = 28_{twenty}$
Historical systems of notation		
3.1431 Egyptian	(20,345)	3.1431 
3.1432 Roman		3.1432 MCMLXX (1970)
3.1433 Babylonian		3.1433  (123)
3.1434 Mayan		3.1434 (143)

3.2 Integral Number System

3.21 Definition of an integer

3.21 Let S be the set of all ordered pairs of whole numbers. Let the relation R on S be: $(a,b)\ R\ (c,d) \leftrightarrow a + d = b + c$. R is an equivalence relation on S that partitions S into mutually exclusive subsets. Any one of these subsets is defined to be an integer.

3.22 Operations

3.221 Addition, a binary operation

3.2211 Definition

3.2211 $\forall a,b \in I$ where $m,n,o,p \in W$ and (m,n) represents a and (o,p) represents b, $a + b = (m,n) \oplus (o,p) = [(m+o), (n+p)]$

3.2212 Properties

3.22121 Closure

3.22121 $\forall a,b \in I \quad (a + b) \in I$

3.22122 Commutative

3.22122 $\forall a,b \in I \quad (a + b) = (b + a)$

3.22123 Associative

3.22123 $\forall a,b,c \in I \quad (a + b) + c = a + (b + c)$

3.22124 Identity

3.22124 $\forall a \in I \quad (a + 0) = a$

3.22125 Inverse

3.22125 $\forall a \in I\ \exists k \in I \ni a + k = 0$

3.2213 Computation and Word Problems

3.222 Subtraction, a binary operation

3.2221 Definition

3.2221 Using the ordered pair approach, $\forall a,b \in I \quad a - b = (m,n) \ominus (o,p) = [(m+p),(n+o)]$. As the inverse of addition, $\forall a,b,c \in I \quad a - b = c \leftrightarrow c + b = a$

3.21 One of
might
and lo

{(...,(
(8,10

3.2211 Cons
-3 and +4
represent
represent
(6,2) =
The order
represent
Thus, -3

3.22121 -5
(-5

3.22122 (-

3.22123 (+
+ 4 + (-

3.22124 (-

3.22125 -7

3.2213 Mr.
$300 at t
$400. Wh

3.2221 -3
[(5+

-5 becs

System of an integer	3.21Let S be the set of all ordered pairs of whole numbers. Let the relation R on S be: (a,b) R (c,d) $\leftrightarrow$ a + d = b + c R is an equivalence relation on S that partitions S into mutually exclusive subsets. Any one of these subsets is defined to be an integer.	3.21 One of these subsets might be the integer -2 and look like the following $\{(\ldots,(5,7), (6,8), (7,9), (8,10), (9,11), \ldots\}$
ion, a binary operation 1 Definition	3.2211 $\forall a,b \in I$ where m,n,o,p $\in W$ and (m,n) represents a and (o,p) represents b, a + b = (m,n) $\oplus$ (o,p) = $\lfloor(m+o), (n+p)\rfloor$	3.2211 Consider the sum of -3 and +4. Let (2,5) represent -3, and let (6,2) represent +4. Then (2,5) $\oplus$ (6,2) = $\lfloor(2+6),(5+2)\rfloor$ = (8,7) The order pair (8,7) represents the integer +1. Thus, -3 + +4 = +1.
2 Properties		
3.22121 Closure	3.22121 $\forall a,b \in I$ (a + b) $\in I$	3.22121 -5,-7 $\in$ I. (-5 + -7) $\in$ I.
3.22122 Commutative	3.22122 $\forall a,b \in I$ (a + b) = (b + a)	3.22122 (-9 + +3) = (+3 + -9)
3.22123 Associative	3.22123 $\forall a,b,c \in I$ (a + b) + c = a + (b + c)	3.22123 (+4 + -2) + -1 = + 4 + (-2 + -1)
3.22124 Identity	3.22124 $\forall a \in I$ (a + 0) = a	3.22124 (-7 + 0) = -7
3.22125 Inverse	3.22125 $\forall a \in I \exists k \in I \ni$ a + k = 0	3.22125 -7 + +7 = 0
3 Computation and Word Problems		3.2213 Mr. X is overdrawn by $300 at the bank. He deposits $400. What is his balance?
action, a binary operation 1 Definition	3.2221 Using the ordered pair approach, $\forall a,b \in I$ a - b = (m,n) $\ominus$ (o,p) = $\lfloor(m+p),(n+o)\rfloor$. As the inverse of addition, $\forall a,b,c \in I$ a - b = c $\leftrightarrow$ c + b = a	3.2221 -3 - +2 = (5,8) $\ominus$ (6,4)= $\lfloor(5+4), (8+6)\rfloor$ = (9,14) = -5. Thus, -3 - +2= -5 because -5 + +2 = -3.

3.2222 Properties-Closure	3.2222 $\forall a,b \in I \quad (a-b) \in I$	3.2222 -4,-6 (-4 -
3.2223 Computation and Word Problems	3.2223	3.2223 -257 -
3.223 Multiplication, a binary operation		
3.2231 Definition	3.2231 $\forall a,b \in I \quad ab = (m,n)(o,p) = \lvert(mo + np), (mp + no)\rvert$	3.2231 -4 x represent -4 +3. Then -4 = |(2 x 4 + = (14,26) =
3.2232 Properties		
3.22321 Closure	3.22321 $\forall a,b \in I \quad (ab) \in I$	3.22321 -9, -9
3.22322 Commutative	3.22322 $\forall a,b \in I \quad (ab) = (ba)$	3.22322 (-11
3.22323 Associative	3.22323 $\forall a,b,c \in I \quad (ab)c = a(bc)$	3.22323 (+1
3.22324 Identity	3.22324 $\forall a \in I \quad (a \times 1) = a$	3.22324 -56
3.22325 Distributive	3.22325 $\forall a,b,c \in I \quad a(b + c) = ab + ac$	3.22325 -1((-1
3.2233 Computation and Word Problems		3.2233 Use t property to for multipli
3.224 Division, a binary operation		
3.2241 Definition	3.2241 $\forall a,b,c \in I \quad a \div b = c \leftrightarrow c \times b = a$	3.2241 -10 ÷ -5 x
3.2242 Computation and Word Problems	3.2242	3.2242 Compu
3.23 Order relation "is less than"		
3.231 Definition	3.231 $\forall a,b \in I \quad (a<b) \leftrightarrow \exists k \neq 0 \in I^{+} \ni a + k = b$	3.231 +2 < +2 +
3.232 Properties		
3.2321 Transitivity	3.2321 $\forall a,b,c \in I \quad (a<b) \wedge (b<c) \rightarrow (a<c)$	3.2321 (-6<-
3.2322 Trichotomy	3.2322 $\forall a,b \in I$ one and only one of the following: $a<b$; $a=b$; $b<a$	3.2322 For i -100<+

3.2222 Properties-Closure	3.2222 $\forall a,b \in I \quad (a-b) \in I$	3.2222 $-4,-6 \in I$ $(-4 - -6) \in I$
3.2223 Computation and Word Problems	3.2223	3.2223 $-257 - +3,457 = -3,714$
Multiplication, a binary operation 3.2231 Definition	3.2231 $\forall a,b \in I \quad ab = (m,n)(o,p) = \lvert(mo + np), (mp + no)\rvert$	3.2231 $-4 \times +3$. Let (2,6) represent -4 and (4,1) represent +3. Then $-4 \times +3 = (2,6) \times (4,1) = \lvert(2 \times 4 + 6 \times 1), (2 \times 1 + 6 \times 4)\rvert = (14,26) = -12$.
3.2232 Properties		
3.22321 Closure	3.22321 $\forall a,b \in I \quad (ab) \in I$	3.22321 $-9,+5 \in I$. $-9 \times +5 \in I$.
3.22322 Commutative	3.22322 $\forall a,b \in I \quad (ab) = (ba)$	3.22322 $(-11 \times +76) = (+76 \times -11)$
3.22323 Associative	3.22323 $\forall a,b,c \in I \quad (ab)c = a(bc)$	3.22323 $(+1 \times -1) \times +3 = +1 \times (-1 \times +3)$
3.22324 Identity	3.22324 $\forall a \in I \quad (a \times 1) = a$	3.22324 $-56 \times 1 = -56$
3.22325 Distributive	3.22325 $\forall a,b,c \in I \quad a(b + c) = ab + ac$	3.22325 $-1(+23 + -11) = (-1 \times +23) + (-1 \times -11)$
3.2233 Computation and Word Problems		3.2233 Use the distributive property to justify the algorithm for multiplication of integers.
Division, a binary operation 3.2241 Definition	3.2241 $\forall a,b,c \in I \quad a \div b = c \leftrightarrow c \times b = a$	3.2241 $-10 \div +2 = -5$ because $-5 \times +2 = -10$
3.2242 Computation and Word Problems	3.2242	3.2242 Compute: $-1,602 \div +9$
relation "is less than" Definition	3.231 $\forall a,b \in I \quad (a<b) \leftrightarrow \exists k \neq 0 \in I \ni a + k = b$	3.231 $+2 < +10$ because $+2 + \underline{+8} = +10$.
Properties 3.2321 Transitivity	3.2321 $\forall a,b,c \in I \quad (a<b) \wedge (b<c) \rightarrow (a<c)$	3.2321 $(-6<-2) \wedge (-2<0) \rightarrow (-6<0)$
3.2322 Trichotomy	3.2322 $\forall a,b \in I$ one and only one of the following: $a<b$; $a=b$; $b<a$	3.2322 For integers -100 and +5, $-100<+5$ is true.

3.3 Rational Number System (an order field)

3.31 Definition of a rational number

3.31 Let S be the set of all ordered pairs of integers. Let the relation R on S be: $(a,b)\ R\ (c,d) \leftrightarrow ad = bc$. R is an equivalence relation on S that partitions S into mutually disjoint subsets. Any one of these subsets is defined to be a rational number.		3.31 One of … might be the … 1/2 and look … … (-3,-… (+1,+…

3.32 Operations

3.321 Addition, a binary operation

3.3211 Definition

$3.3211\ \forall a/b, c/d \in Ra\ a/b + c/d = (ad + bc)/bd$	3.3211 2/3 + …

3.3212 Properties

3.32121 Closure	$3.32121\ \forall a/b, c/d \in Ra (a/b + c/d) \in Ra$
3.32122 Commutative	$3.32133\ \forall a/b, c/d \in Ra\ (a/b + c/d) = (c/d + a/b)$
3.32123 Associative	$3.32123\ \forall a/b, c/d, e/f \in Ra\ (a/b + c/d) + e/f = a/b + (c/d + e/f)$
3.32124 Identity	$3.32124\ \forall a/b \in Ra\ \ a/b + 0/1 = a/b$
3.32125 Inverse	$3.32125\ \forall a/b \in Ra\ \exists e/f \in Ra \ni a/b + e/f = 0/1$

3.32121	2/3 … of t…
3.32133	4/5 …
3.32123	(8/9 … 8/9 …
3.32124	11/1…
3.32125	-4/5…

3.3213 Computation and Word Problems

3.32131 Equal denominators	3.32131 1/4 …
3.32132 Unequal denominators	3.32132 1/2 …
3.32133 Decimal notation	3.32133 1.09…
3.32134 Mixed notation	3.32134 4 ½ …

3.322 Subtraction, a binary operation

3.3221 Definition

$\forall a/b, c/d, e/f \in Ra\ a/b - c/d = e/f \leftrightarrow e/f + c/d = a/b$	3.3221 3/8 … because 1/4 …

3.3222 Properties-Closure

$3.3222\ \forall a/b,\ c/d \in Ra\ (a/b - c/d) \in Ra$	3.3222 8/9, (8/9 …

tem (an order field) a rational number	3.31 Let S be the set of all ordered pairs of integers. Let the relation R on S be: (a,b) R (c,d) $\leftrightarrow$ ad =bc. R is an equivalence relation on S that partitions S into mutually disjoint subsets. Any one of these subsets is defined to be a rational number.	3.31 One of these subsets might be the rational number 1/2 and look like the following: $\{\ldots (-3,-6), (-2,-4), (-1,-2), (+1,+2), (+2,+4), \ldots\}$
, a binary operation efinition	3.3211 $\forall$ a/b,c/d $\in$ Ra a/b + c/d= (ad + bc)/bd	3.3211 $2/3 + 3/4 = \frac{(2\times4 + 3 \times 3)}{3 \times 4} = \frac{8 + 9}{12} = 17/12$
roperties .32121 Closure	3.32121 $\forall$ a/b,c/d $\in$ Ra(a/b + c/d) $\in$ Ra	3.32121 2/3 + 3/4 is a member of the set of rationals
.32122 Commutative	3.32133 $\forall$ a/b,c/d $\in$ Ra (a/b + c/d) = (c/d + a/b)	3.32133 4/5 + 1/2 = 1/2 + 4/5
.32123 Associative	3.32123 $\forall$ a/b,c/d,e/f $\in$ Ra (a/b + c/d) + e/f = a/b + (c/d + e/f)	3.32123 (8/9 + 3/5) + 2/3 = 8/9 + (3/5 + 2/3)
.32124 Identity	3.32124 $\forall$ a/b $\in$ Ra a/b + 0/1 = a/b	3.32124 11/12 + 0/1 = 11/12
.32125 Inverse	3.32125 $\forall$ a/b $\in$ Ra $\exists$ e/f $\in$ Ra $\ni$ a/b + e/f = 0/1	3.32125 -4/5 + +4/5= 0/1
omputation and Word Problems .32131 Equal denominators .32132 Unequal denominators .32133 Decimal notation .32134 Mixed notation		3.32131 1/4 + 5/4 = 6/4 3.32132 1/2 + 2/3 = 7/6 3.32133 1.09 + 3.89 = 4.98 3.32134 $4\frac{1}{2}$ + $19\frac{1}{4}$ = 23 3/4
ion, a binary operation efinition	$\forall$ a/b,c/d,e/f $\in$ Ra a/b - c/d = e/f $\leftrightarrow$ e/f + c/d = a/b	3.3221 3/8 - 1/8 = 1/4 because 1/4 + 1/8 = 3/8
roperties-Closure	3.3222 $\forall$ a/b, c/d $\in$ Ra (a/b - c/d) $\in$ Ra	3.3222 8/9, 7/8 $\in$ Ra (8/9 - 7/8) $\in$ Ra

3.3223 Computation and Word Problems	
3.32231 Equal denominators	3.32231 A cup is filled 3/4 mark. I remove 1 How much liquid rema
3.32232 Unequal denominators	3.32232 The difference 2/5 cups and 4/9 cup
3.32233 Decimal notation	3.32233 11.6 - 5.7 =
3.32234 Mixed notation	3.32234 $5\frac{1}{2} - 3\frac{1}{4} = 2$
3.323 Multiplication, a binary operation	
3.3231 Definition	3.3231 $\forall a/b, c/d \in Ra$ $a/b \times c/d = ac/bd$
3.3232 Properties	
3.32321 Closure	3.32321 $\forall a/b, c/d \in Ra$ $(a/b \times c/d) \in Ra$
3.32322 Commutative	3.32322 $\forall a/b, c/d \in Ra$ $(a/b \times c/d) = (c/d \times a/b)$
3.32323 Associative	3.32323 $\forall a/b, c/d, e/f \in Ra$ $(a/b \times c/d) \times e/f = a/b \times (c/d \times e/f)$
3.32324 Distributive	3.32324 $\forall a/b, c/d, e/f \in Ra$ $a/b \times (c/d + e/f) = (a/b \times c/d) + (a/b \times e/f)$
3.32325 Identity	3.32325 $\forall a, b \in Ra$ $a/b \times 1/1 = a/b$
3.32326 Inverse	3.32326 $\forall a/b \neq 0/1 \in Ra$ $\exists e/f \in Ra$ $\ni a/b \times c/d = 1/1$
3.3233 Computation and Word Problems	
3.32331 Fraction notation	3.32331 Compute: 3/4
3.32332 Decimal notation	3.32332 Compute: 3.14
3.32333 Mixed notation	3.32333 John has $3\frac{1}{2}$ ti as much money as Mary. $1.75. How much money
3.324 Division, a binary operation	
3.3241 Definition	3.3241 $\forall a/b, c/d \neq 0, e/f \in Ra$ $a/b \div c/d = e/f \leftrightarrow e/f \times c/d = a/b$
3.3242 Properties-Closure	3.3242 $\forall a/b, c/d \neq 0 \in Ra$ $(a/b \div c/d) \in Ra$

ɛation and Word Problems		
31 Equal denominators		3.32231 A cup is filled to the 3/4 mark. I remove 1/4 liquid. How much liquid remains?
32 Unequal denominators		3.32232 The difference between 2/5 cups and 4/9 cups is __?
33 Decimal notation		3.32233 11.6 - 5.7 = ___.
34 Mixed notation		3.32234 $5\frac{1}{2} - 3\frac{1}{4} = 2\frac{1}{4}$
ion, a binary operation		
tion	3.3231 $\forall$ a/b,c/d ∈ Ra a/b x c/d = ac/bd	3.3231 5/8 x 6/7 = 30/56
rties		
21 Closure	3.32321 $\forall$ a/b,c/d ∈ Ra (a/b x c/d) ∈ Ra	3.32321 1/2,3/4 ∈ Ra (1/2 x 3/4) ∈ Ra
22 Commutative	3.32322 $\forall$ a/b, c/d ∈ Ra (a/b x c/d) = (c/d x a/b)	3.32322 (2/3 x 4/5) = (4/5 x 2/3)
23 Associative	3.32323 $\forall$ a/b,c/d,e/f ∈ Ra (a/b x c/d) x e/f = a/b x (c/d x e/f)	3.32323 (5/6 x7/8) x 9/10 = 5/6 x (7/8 x 9/10)
24 Distributive	3.32324 $\forall$ a/b, c/d, e/f ∈ Ra a/b x(c/d + e/f) = (a/b x c/d) + (a/b x e/f)	3.32324 1/2 x (5/6 + 7/8) = 1/2 x 5/6 + 1/2 x 7/8
25 Identity	3.32325 $\forall$ a,b ∈ Ra a/b x 1/1 = a/b	3.32325 5/6 x 1/1 = 5/6
26 Inverse	3.32326 $\forall$ a/b≠0/1 ∈ Ra $\exists$ e/f∈Ra ∋ a/b x c/d = 1/1	3.32326 -6/7 x -7/6 = 1/1
tation and Word Problems		
31 Fraction notation		3.32331 Compute: 3/4 x 2/6 = __
32 Decimal notation		3.32332 Compute: 3.14 x 8.132 =
33 Mixed notation		3.32333 John has $3\frac{1}{2}$ times as much money as Mary. Mary has $1.75. How much money has John?
binary operation		
ition	3.3241 $\forall$ a/b, c/d≠0,e/f ∈ Ra a/b ÷ c/d = e/f ⟷ e/f xc/d = a/b	3.3241 4/5÷2/3 = 12/10 because 12/10 x 2/3 = 4/5
rties-Closure	3.3242 $\forall$ a/b, c/d≠0∈Ra (a/b ÷c/d) ∈ Ra	3.3242 5/6,7/8 ∈ Ra (5/6÷7/8) ∈ Ra

3.3243 Computation and Word Problems
3.32431 Fraction notation
3.32432 Decimal notation
3.32433 Mixed notation

3.33 Order relation "is less than"

3.331 Definition

3.332 Properties

3.3321 Transitivity

3.3322 Trichotomy

3.34 Numeration (names of rational numbers)

3.341 Equivalent fraction notation

3.342 Equivalent mixed notation

3.343 Equivalent fraction notation with terminating decimals

3.344 Equivalent fraction notation with repeating decimals

3.345 Equivalent per cent notation

3.3451 Definition of per cent

3.3452 Ratio and proportion

3.3453 Computations related to percent, ratio, and proportion

3.346 Expanded notation

3.4 Finite Number Systems

3.41 Definition of a finite system (mod m, where m is an integer)

	3.32431 Com
	3.32432 Com
	3.32433 Com
3.331 $\forall a/b, c/d \in Ra$ $a/b < c/d \leftrightarrow \exists e/f \in Ra^{+} \ni a/b + e/f = c/d$	3.331 1/2 1/2 + 1/
3.3321 $\forall a/b, c/d, e/f \in Ra$ $a/b < c/d \wedge c/d < e/f \rightarrow a/b < e/f$	3.3321 -2 +1/4 <
3.3322 $\forall a/b, c/d \in Ra$ one and only one of the following is true: $a/b < c/d$; $a/b = c/d$; $c/d < a/b$	3.3322 1/ 1/
	3.341 1/2=2
	3.342 3 $\frac{1}{2}$ =
	3.343 1/8
	3.344 4/7
3.3451 per cent means per hundred	3.3451 1/4
	3.3452 x:3 x
	3.3453 Mik of 55% g right on many ite
3.41 Let S be the set of integers. Let the relation R on S be: $\forall a,b \in I$ $a \equiv b \pmod m \leftrightarrow m \mid (a-b)$. R is an equivalence relation on S that partitions S into mutually disjoint subsets. Depending on m, your equivalence classes are the "numbers" of your finite	3.41 For ex equivalenc 0_7, 1_7, 2_7 The subset by 0_7 woul +

utation and Word Problems		
431 Fraction notation		3.32431 Compute: 3/4 ÷ 1/3 = 9/4
432 Decimal notation		3.32432 Compute: 24.67 ÷ 2.12= ___
433 Mixed notation		3.32433 Compute: $4\frac{1}{4} \div 2\frac{1}{2}$ = ___
s less than"	3.331 $\forall a/b, c/d \in Ra$ $a/b < c/d \leftrightarrow \exists_{e/f \in Ra^+} a/b + e/f = c/d$	3.331 1/2 < 3/4 because 1/2 + 1/4 = 3/4
sitivity	3.3321 $\forall a/b, c/d, e/f \in Ra$ $a/b < c/d \wedge c/d < e/f \rightarrow a/b < e/f$	3.3321 -2/4 < +1/4 and +1/4 < +1/2 → -2/4 < + 1/2
hotomy	3.3322 $\forall a/b, c/d \in Ra$ one and only one of the following is true: $a/b < c/d$; $a/b = c/d$; $c/d < a/b$	3.3322 1/2, 4/5 ∈ Ra; 1/2 < 4/5 is true.
of rational numbers)		
fraction notation		3.341 1/2=2/4=3/6=4/8=...
mixed notation		3.342 $3\frac{1}{2} = 7/2$
fraction notation with decimals		3.343 1/8 = 0.125
fraction notation with ecimals		3.344 4/7 = $0.\overline{571428}$
per cent notation		
nition of per cent	3.3451 per cent means per hundred	3.3451 1/4 would mean 25%
o and proportion		3.3452 x:3:: 10:6 x = 5
utations related to ent, ratio, and proportion		3.3453 Mike received a score of 55% getting 40 answers right on a math test. How many items were on the test?
notation		
inite system (mod m, where m is an integer)	3.41 Let S be the set of integers. Let the relation R on S be: $\forall a,b \in I$ $a \equiv b \pmod m \leftrightarrow m \mid (a-b)$. R is an equivalence relation on S that partitions S into mutually disjoint subsets. Depending on m, your equivalence classes are the "numbers" of your finite system.	3.41 For example, in mod 7, your equivalence classes would be $0_7, 1_7, 2_7, 3_7, 4_7, 5_7, 6_7$. The subset of integers represented by 0_7 would be: ...,-14,-7,0, +7, +14, ...

3.42 Operations		
3.421 Addition, a binary operation		
3.4211 Definition	$3.4211\ \forall a_m, b_m \in I_m \quad a_m \oplus b_m = (a + b)_m$	3.4211 4_5
3.4212 Properties		
3.42121 Closure	$3.42121\ \forall a_m, b_m \in I_m \quad (a_m \oplus b_m) \in I_m$	3.42121
3.42122 Commutative	$3.42122\ \forall a_m, b_m \in I_m \quad (a_m \oplus b_m) = (b_m \oplus a_m)$	3.42122
3.42123 Associative	$3.42123\ \forall a_m, b_m, c_m \in I_m \quad (a_m \oplus b_m) \oplus c_m = a_m \oplus (b_m \oplus c_m)$	3.42123(
3.42124 Identity	$3.42124\ \forall a_m \in I_m \quad a_m \oplus 0_m = a_m$	3.42124
3.42125 Inverse	$3.42125\ \forall a_m \in I_m \ \exists c_m \in I_m \ni a_m \oplus c_m = 0_m$	3.42125
3.422 Multiplication, a binary operation		
3.4221 Definition	$3.4221\ \forall a_m, c_m \in I_m \quad a_m \odot c_m = (ac)_m$	3.4221 2
3.4222 Properties		
3.42221 Closure	$3.42221\ \forall a_m, b_m \in I_m \quad (a_m \odot b_m) \in I_m$	3.42221
3.42222 Associative	$3.42222\ \forall a_m, b_m, c_m \in I_m \quad (a_m \odot b_m) \odot c_m = a_m \odot (b_m \odot c_m)$	3.42222
3.42223 Identity	$3.42223\ \forall a_m \in I_m \quad a_m \odot 1_m = a_m$	3.42223
3.42224 Commutative	$3.42224\ \forall a_m, b_m \in I_m \quad (a_m \odot b_m) = (b_m \odot a_m)$	3.42224
3.42225 Distributive	$3.42225\ \forall a_m, b_m, c_m \in I_m \quad a_m \odot (b_m \oplus c_m) = (a_m \odot b_m) \oplus (a_m \odot c_m)$	3.42225 $(2_6 \odot 4_6)$
3.43 Applications		
3.431 Divisibility Tests		3.431 Do
3.432 Addition checks, i.e. casting out nines		3.432 54 15 ou
3.433 Linear Diophantine equations		3.433 So

binary operation nition	3.4211 $\forall a_m, b_m \in I_m$ $a_m \oplus b_m = (a + b)_m$	3.4211 $4_5 \oplus 3_5 = 2_5$
erties		
21 Closure	3.42121 $\forall a_m, b_m \in I_m$ $(a_m \oplus b_m) \in I_m$	3.42121 $4_6 \oplus 8_6 \in I_6$
22 Commutative	3.42122 $\forall a_m, b_m \in I_m$ $(a_m \oplus b_m) = (b_m \oplus a_m)$	3.42122 $(100_4 \oplus 72_4) = (72_4 \oplus 100_4)$
23 Associative	3.42123 $\forall a_m, b_m, c_m \in I_m$ $(a_m \oplus b_m) \oplus c_m = a_m \oplus (b_m \oplus c_m)$	3.42123 $(4_7 \oplus 3_7) \oplus 5_7 = 4_7 \oplus (3_7 \oplus 5_7)$
24 Identity	3.42124 $\forall a_m \in I_m$ $a_m \oplus 0_m = a_m$	3.42124 $8_9 \oplus 0_9 = 8_9$
25 Inverse	3.42125 $\forall a_m \in I_m \exists c_m \in I_m$, $a_m \oplus c_m = 0_m$	3.42125 $2_5 \oplus 3_5 = 0_5$
ion, a binary operation nition	3.4221 $\forall a_m, c_m \in I_m$ $a_m \odot c_m = (ac)_m$	3.4221 $2_7 \odot 4_7 = (8_7) = 1_7$
erties		
221 Closure	3.42221 $\forall a_m, b_m \in I_m$ $(a_m \odot b_m) \in I_m$	3.42221 $(3_9 \odot 7_9) \in I_9$
222 Associative	3.42222 $\forall a_m, b_m, c_m \in I_m$ $(a_m \odot b_m) \odot c_m = a_m \odot (b_m \odot c_m)$	3.42222 $(2_7 \odot 3_7) \odot 4_7 = 2_7 \odot (3_7 \odot 4_7)$
223 Identity	3.42223 $\forall a_m \in I_m$ $a_m \odot 1_m = a_m$	3.42223 $4_8 \odot 1_8 = 4_8$
224 Commutative	3.42224 $\forall a_m, b_m \in I_m$ $(a_m \odot b_m) = (b_m \odot a_m)$	3.42224 $(3_5 \odot 4_5) = (4_5 \odot 3_5)$
225 Distributive	3.42225 $\forall a_m, b_m, c_m \in I_m$ $a_m \odot (b_m \oplus c_m) = (a_m \odot b_m) \oplus (a_m \odot c_m)$	3.42225 $2_6 \odot (4_6 \oplus 5_6) = (2_6 \odot 4_6) \oplus (2_6 \odot 5_6)$
y Tests ecks, i.e. casting out nines		3.431 Does $9 \mid 603$? 3.432 547 + 89 + 930 = 1566. Check by casting out nines.
nantine equations		3.433 Solve: $7x + 16y = 40$

4. Number Theory
4.1 Odds and Evens

4.1 Def. The integer b divides the integer a ($b|a \leftrightarrow \exists$ an integer $k \ni a = bk$.
An even integer, therefore, is any integer, d, such that $2|d$. An odd integer is any integer that is not even.

4.2 Factors and primes

4.2 An integer b is a factor of integer a $\leftrightarrow b|a$.
$\forall p \in I^+$ p is prime $\leftrightarrow p \neq 1$ and the only positive factors of p are 1 and p.

4.3 Composite numbers

4.3 $\forall c \in I$, c is a composite number $\leftrightarrow c \neq 1$ and c is not prime.

4.4 Multiples

4.4 $\forall a,b \in I$ a is a multiple of b $\leftrightarrow$ b is a factor of a.

4.5 Greatest Common Factor (GCF)

4.5 The greatest number that is a member of the set of common factors of the positive integers a and b is the GCF.

4.6 Least common multiple (LCM)

4.6 The positive integer c is the LCM of positive integers a and b $\leftrightarrow$ c is a multiple of a c is a multiple of b c is the least multiple.

4.7 Prime factorization

4.7 If a is any composite, then a may be written as a product of primes.

4.8 Relative primes

4.8 The positive integers a and b are relatively prime $\leftrightarrow$ the GCF of a and b is 1.

4.9 Number patterns
4.91 Arithmetic progressions

4.91 An arithmetic progression is a succession of numbers in which there is a common difference between each member and its successor.

4.1 0 = {...,-
E = {...,-

4.2 The factors
1,2,3,4,6,
7 is a pri
its only f

4.3 12 is a co

4.4 The set of
0,3,6,9,1

4.5 The GCF of

4.6 The LCM of

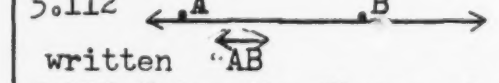
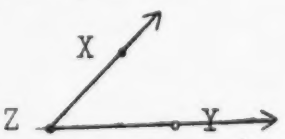
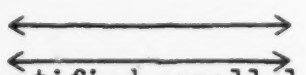
4.7 The prime
12 = 2 x

4.8 3 and 5 a
because t
is 1.

4.91 1,3,5,7,
arithmet

	4.1 Def. The integer $\underline{b}$ divides the integer $\underline{a}$ $(b\mid a) \leftrightarrow \exists$ an integer $\underline{k}$ ∋ $a = bk$. An even integer, therefore, is any integer, $\underline{d}$, such that $2\mid d$. An odd integer is any integer that is not even.	4.1 $O = \{\ldots,-3,-1,+1,+3, \ldots\}$ $E = \{\ldots,-4,-2,0,+2,+4, \ldots\}$
	4.2 An integer $\underline{b}$ is a factor of integer $\underline{a} \leftrightarrow b\mid a$. $\forall p \in I^+$ p is prime $\leftrightarrow p \neq 1$ and the only positive factors of p are 1 and p.	4.2 The factors of 24 are: 1,2,3,4,6,8,12,24 . 7 is a prime number because its only factors are 1 and 7.
	4.3 $\forall c \in I$, $\underline{c}$ is a composite number $\leftrightarrow$ $c \neq 1$ and $\underline{c}$ is not prime.	4.3 12 is a composite number.
	4.4 $\forall a,b \in I$ $\underline{a}$ is a multiple of $\underline{b} \leftrightarrow \underline{b}$ is a factor of $\underline{a}$.	4.4 The set of multiples of 3 are: 0,3,6,9,12,15,18,21,...
tor (GCF)	4.5 The greatest number that is a member of the set of common factors of the positive integers $\underline{a}$ and $\underline{b}$ is the GCF.	4.5 The GCF of 12 and 8 is 4.
le (LCM)	4.6 The positive integer $\underline{c}$ is the LCM of positive integers $\underline{a}$ and $\underline{b} \leftrightarrow \underline{c}$ is a multiple of $\underline{a}$ $\underline{c}$ is a multiple of $\underline{b} \wedge$ c is the least multiple.	4.6 The LCM of 12 and 8 is 24.
	4.7 If $\underline{a}$ is any composite, then $\underline{a}$ may be written as a product of primes.	4.7 The prime factorization of 12: $12 = 2 \times 2 \times 3$
	4.8 The positive integers $\underline{a}$ and $\underline{b}$ are relatively prime $\leftrightarrow$ the GCF of $\underline{a}$ and $\underline{b}$ is 1.	4.8 3 and 5 are relatively prime because the GCF of 3 and 5 is 1.
gressions	4.91 An arithmetic progression is a succession of numbers in which there is a common difference between each member and its successor.	4.91 1,3,5,7,9,11,... is an arithmetic progression

4.92 Geometric progressions	4.92 A geometric progression is a succession of numbers in which there is a common ratio between each member and its successor.	4.92 1,2,4,8,16 is a geome
4.93 Triangular numbers		4.93 3(..),6(
4.94 Square numbers		4.94 4(..), 9
4.95 Factorial numbers	4.95 $\forall n \in I^+$ $n! = n \times (n-1) \times (n-2) \times \dots \times 1$.	4.95 7! = 7 x 6
4.96 Fibonacci numbers	4.96 $F_{n+2} = F_{n+1} + F_n$, $n \geq 1$	4.96 1,1,2,3,5, is th
5. Geometry		
5.1 Non-metric		
5.11 Development of geometric concepts		
5.111 point and betweenness	5.111 undefined	5.111 Often pic to locate a spe
5.112 Line	5.112 undefined	5.112 .A written $\overleftrightarrow{AB}$
5.113 Segment	5.113 The set of points consisting of two end points and all points between.	5.113 Using the we write
5.114 Ray	5.114 Consider a line. Select any point on that line. A ray would consist of that point and all points in only one direction.	5.114 Using the we write $\overrightarrow{BA}$ However, $\overrightarrow{BA} \neq$
5.115 Angle	5.115 Two rays that have common endpoints and that are not equal.	5.115 X Z we write, ∠X
5.116 Related lines: intersecting, skew, parallel, etc.		5.116 To unidentifi
5.117 Regions: interior, exterior	5.117 A simple closed curve partitions the plane into 3 sets: 1)the curve itself, 2)the interior of the curve, and 3)the exterior of the curve.	5.117 .B Point B is co interior, whi exterior.

ic progressions	4.92 A geometric progression is a succession of numbers in which there is a common ratio between each member and its successor.	4.92 1,2,4,8,16,32,64,128,... is a geometric progression.
lar numbers		4.93 3, 6, 10, ...
numbers		4.94 4, 9, 16
al numbers	4.95 $\forall n \epsilon I^+$ $n! = n \times (n-1) \times (n-2) \times \ldots \times 1$.	4.95 $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
ci numbers	4.96 $F_{n+2} = F_{n+1} + F_n$, $n \geq 1$	4.96 1,1,2,3,5,8,13,21,34,55,... is the set of Fibonacci numbers
ent of geometric concepts int and betweenness	5.111 undefined	5.111 Often pictured with a [.] to locate a specific place.
ne	5.112 undefined	5.112  written $\overleftrightarrow{AB}$
gment	5.113 The set of points consisting of two end points and all points between.	5.113 Using the figure of 5.112, we write $\overleftrightarrow{AB}$.
y	5.114 Consider a line. Select any point on that line. A ray would consist of that point and all points in only one direction.	5.114 Using the figure of 5.112, we write $\overrightarrow{BA}$ or $\overrightarrow{AB}$. However, $\overrightarrow{BA} \neq \overrightarrow{AB}$.
gle	5.115 Two rays that have common endpoints and that are not equal.	5.115  we write, $\angle XZY$ or $\angle YZX$.
lated lines: intersecting, ew, parallel, etc.		5.116  To unidentified parallel lines.
gions: interior, exterior	5.117 A simple closed curve partitions the plane into 3 sets: 1)the curve itself, 2)the interior of the curve, and 3)the exterior of the curve.	5.117 Point B is contained in the interior, while A is in the exterior.

5.118 Planes and two-dimensional figures

5.1181 Curves

5.11811 Simple, closed (open)

5.11811 A closed curve that does not intersect itself at any point is a simple, closed curve.

5.11811

5.11812 Convex, concave

5.11812 A convex, closed curve is a curve that, when selecting any two points in the interior of the curve, contains all possible segments within the interior.

5.11812

5.1182 Polygons

5.11821 Triangles (Equilateral, Isosceles, Scalene)

5.11821 The union of three rays such that any two rays have a common end-point.

5.11821

5.11822 Quadrilaterals

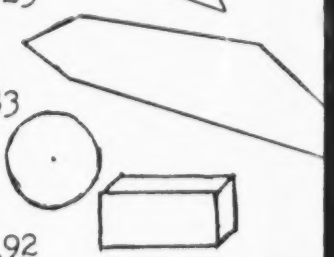
5.11822 A closed union of four segments.

5.11822

5.11823 Other polygons

5.11823 (such as pentagons, hexagons, heptagons, octagons, nonagons, etc.)

5.11823



5.1183 Circles

5.1183 The set of points a specified distance from a given point.

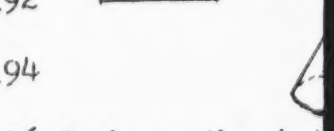
5.1183

5.119 Space and three-dimensional figures

5.1191 Pyramid

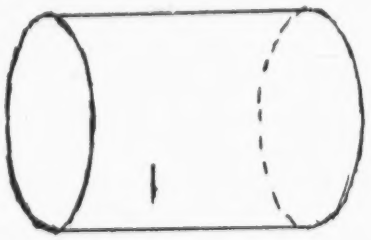
5.1192 Prism

5.1192



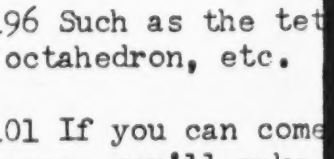
5.1193 Cylinder

5.1193



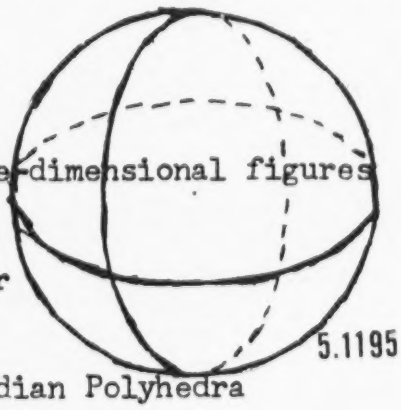
5.1194 Cone

5.1194



5.1195 Sphere

5.1195



5.1196 Archimedian Polyhedra

5.1196 Such as the tet
the octahedron, etc.

5.110 Constructions

5.1101 One-dimensional

5.1101 If you can come
this one, you'll make

5.1102 Two-dimensional

5.1102 e.g. Bisect the given segment.

5.1103 Three-dimensional

5.1103 Construct an ico

5.1191

Planes and two-dimensional figures

5.1181 Curves

5.11811 Simple, closed (open)

5.11811 A closed curve that does not intersect itself at any point is a simple, closed curve.

5.11811

5.11812 Convex, concave

5.11812 A convex, closed curve is a curve that, when selecting any two points in the interior of the curve, contains all possible segments within the interior.

5.11812

5.1182 Polygons

5.11821 Triangles (Equilateral, Isosceles, Scalene)

5.11821 The union of three rays such that any two rays have a common end-point.

5.11821

5.11822 Quadrilaterals

5.11822 A closed union of four segments.

5.11822

5.11823 Other polygons

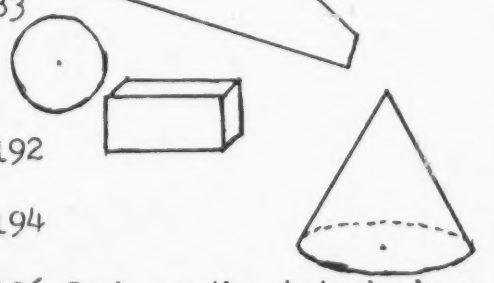
5.11823 (such as pentagons, hexagons, heptagons, octagons, nonagons, etc.)

5.11823

5.1183 Circles

5.1183 The set of points a specified distance from a given point.

5.1183



Space and three-dimensional figures

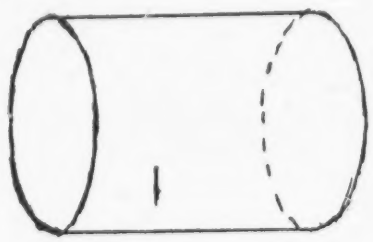
5.1191 Pyramid

5.1192 Prism

5.1192

5.1193 Cylinder

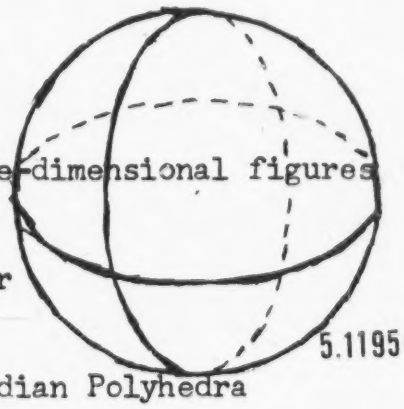
5.1193



5.1194 Cone

5.1194

5.1195 Sphere



5.1195

5.1196 Archimedian Polyhedra

5.1196 Such as the tetrahedron, the octahedron, etc.

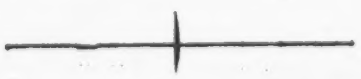
Constructions

5.1101 One-dimensional

5.1101 If you can come up with this one, you'll make a bundle.

5.1102 Two-dimensional

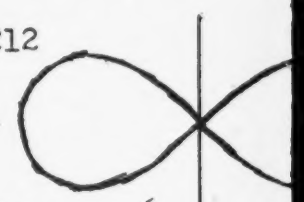
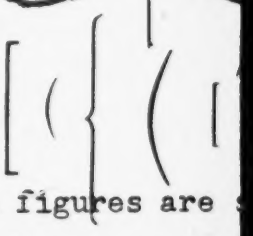
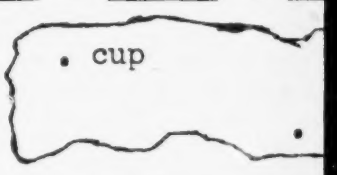
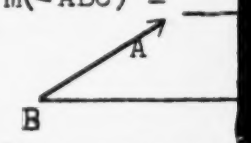
5.1102 e.g. Bisect the given segment.



5.1103 Three-dimensional

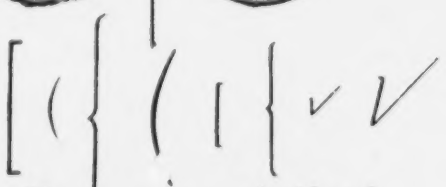
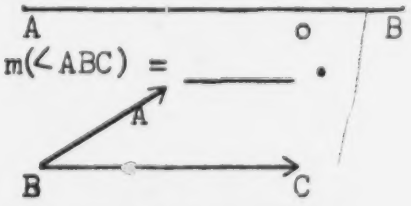
5.1103 Construct an icosahedron.

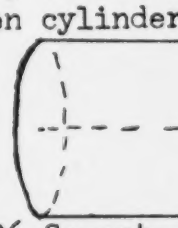
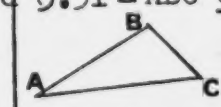
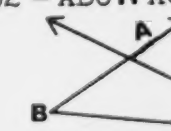
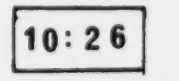
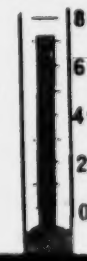
5.1191

5.2 Metric 5.21 Comparing size, shape, and distance 5.211 Congruency	5.211 Two geometric figures are congruent ↔ one figure can be superimposed on the other so that they coincide exactly.	5.211 $\sum \int \Sigma$ Which figures are
5.212 Symmetry	5.212 The correspondence, in size, form, and arrangement, of geometric figures on opposite sides of a plane, line, or point.	5.212 
5.213 Similarity (including similarity and ratio in a right triangle)	5.213 Two figures are similar ↔ they have the same shape; that is, their corresponding parts are proportional and their corresponding angles equal.	5.213  Which figures are
5.214 Scale Drawing		5.214 The actual di cup to saucer is _____  Scale: 1/5" = 10 m
5.215 Pythagorean relation	5.215 $a^2 + b^2 = c^2$	
5.22 Measurement (metric or British) 5.221 Segments to nearest unit		5.221 $m(\overline{AB})$ = _____
5.222 Angles to nearest part of degree		5.222 $m(\angle ABC)$ = _____ 
5.223 Perimeter and circumference	5.223 The perimeter of any geometric figure is the measure of the distance around the figure.	5.223 The perimeter circle to the near _____°

shape, and distance

y

	5.211 Two geometric figures are congruent ↔ one figure can be superimposed on the other so that they coincide exactly.	5.211 $\sum \int \Sigma \int \sum \vee$ Which figures are congruent?
	5.212 The correspondence, in size, form, and arrangement, of geometric figures on opposite sides of a plane, line, or point.	5.212
y (including similarity in a right triangle)	5.213 Two figures are similar ↔ they have the same shape; that is, their corresponding parts are proportional and their corresponding angles equal.	5.213  Which figures are similar?
wing		5.214 The actual distance from cup to saucer is _____ miles. Scale: 1/5" = 10 miles.
rean relation	5.215 $a^2 + b^2 = c^2$	
metric or British) to nearest unit		5.221 m($\overline{AB}$) = _____ cm. (nearest cm.)
o nearest part of degree		5.222 m($\angle ABC$) = _____ . 
r and circumference	5.223 The perimeter of any geometric figure is the measure of the distance around the figure.	5.223 The perimeter of the circle to the nearest ½ inch is _____ .

5.224 Area of plane figures	5.224 Many formulas are available here, such as, the area of a square is equal to the square of the measure of one of its sides.	5.224 The area the nearest $\frac{1}{2}$ s
5.225 Volume of Solids	5.225 Here too, many formulas are available such as the familiar volume of a sphere as $\frac{4}{3}\pi r^3$. However, children should be discouraged from simple memorization of these formulas.	5.225 What is t given cylinder 
5.226 Approximate nature of measurement (greatest possible error)	5.226 The accuracy of a measure is designated by the number of significant digits of the measure. The number of significant digits is the number of digits which specify the number of units in the measurement.	5.226 Compute t approximate num 3.62. Compute approximate nu 8.72. (Sum=55.
5.227 Equivalent measures		**5.227 3 yds.** $\overline{=}$ **m**
5.3 Operations with Geometric figures		
5.31 Union	5.31 (usual definition applied to geometric figures)	5.31 $\Delta ABC \cup$ 
5.32 Intersection	5.32 (usual definition but applied to geometric figures.	5.32 $\angle ABC \cap \overleftrightarrow{AC}$ 
5.4 Measurement of non-geometric quantities		
5.41 Historical units of measure		5.41 span, dig
5.42 Money		5.42 $4.58 in
5.43 Time		5.43 What time clock? 
5.44 Temperature: Fahrenheit and centigrade	5.44 F = 9/5 C + 32	5.44 What temp Centigrade t 

rea of plane figures

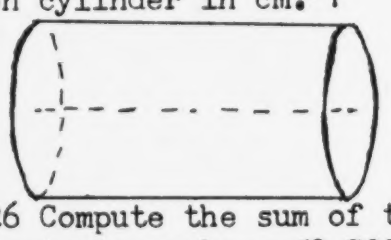
5.224 Many formulas are available here, such as, the area of a square is equal to the square of the measure of one of its sides.

5.224 The area of the square to the nearest $\frac{1}{2}$ sq. in. = _____.

olume of Solids

5.225 Here too, many formulas are available such as the familiar volume of a sphere as $\frac{4}{3}\pi r^3$. However, children should be discouraged from simple memorization of these formulas.

5.225 What is the volume of the given cylinder in cm.^{3}?



pproximate nature of measurement greatest possible error)

5.226 The accuracy of a measure is designated by the number of significant digits of the measure. The number of significant digits is the number of digits which specify the number of units in the measurement.

5.226 Compute the sum of the approximate numbers 52.709 and 3.62. Compute the product of the approximate numbers 11.03 and 8.72. (Sum=55.33; product=96.2)

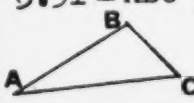
Equivalent measures

5.227 3 yds. $\overset{m}{=}$ 9 ft.

th Geometric figures

5.31 (usual definition applied to geometric figures)

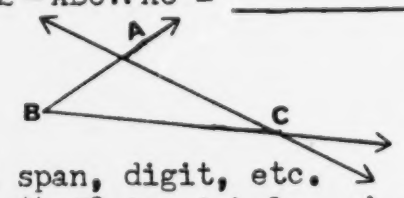
5.31 $\Delta ABC \cup \overline{EF}$ = _____



ction

5.32 (usual definition but applied to geometric figures.

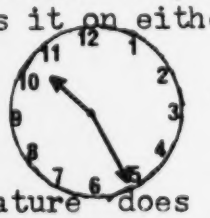
5.32 $\angle ABC \cap \overleftrightarrow{AC}$ = _____



f non-geometric quantities
cal units of measure

5.41 span, digit, etc.
5.42 $4.58 in nickels and pennies
5.43 What time is it on either clock?

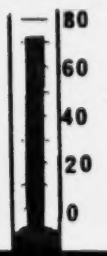
10:26



ture: Fahrenheit and centigrade

5.44 F = 9/5 C + 32

5.44 What temperature does the Centigrade thermometer report?



5.45 Denominate numbers

5.46 Weight, volume, and quantity measures

5.5 Topology (popularly, rubber-sheet geometry)

6. Organizing and Reporting Data

6.1 Graphs

6.11 Equalities and inequalities on the number line

6.12 Ordered pairs on the coordinate plane

6.13 Solution sets of equalities and inequalities on the coordinate plane

6.2 Statistics

6.21 Frequency tables and charts

6.22 Graphs (bar, line, circle, dot, picture)

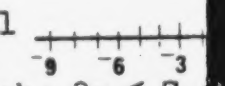
6.221 Construction

5.45 The measure of a quantity named as a sum of two or more numerals in different bases.

5.45 3 yds. 2 f
+ 7 yds. 1 f

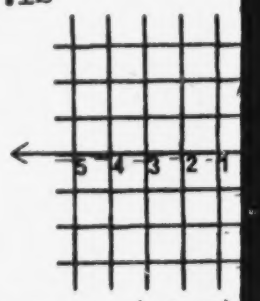
5.46 pints, qua
peck, etc.

6.11



Graph: $3x < 7$

6.12



Graph: (-3,2)

6.13 Using the
of 6.12, graph
for $\{(x,y)$: 2x

6.21 Construct
for this set of
(Scores by fem
on the MNA)
4,14,25,3,7,16,
19,18, 21,22,16
27,26,25,24,23,

6.221 Construct
for the followi
1950- 12 b
1955- 16 b
1960- 21 b
1965- 25 b
Students atter
High School, N

numbers

ume, and quantity measures
ly, rubber-sheet geometry)
ting Data

and inequalities on the

rs on the coordinate plane

ts of equalities and
s on the coordinate plane

tables and charts

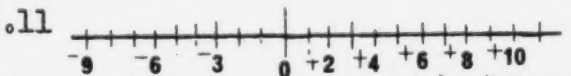
r, line, circle, dot, picture)
truction

5.45 The measure of a quantity named as a sum of two or more numerals in different bases.

5.45 3 yds. 2 ft. 11in.
+ 7 yds. 1 ft. 9 in.

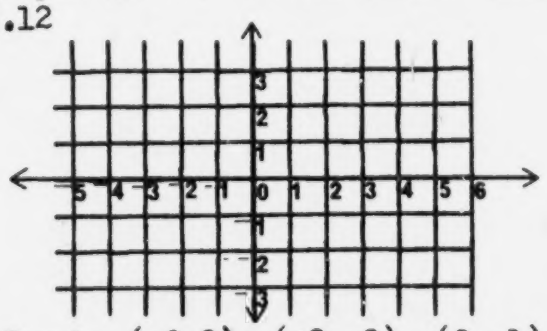
5.46 pints, quarts, pounds, peck, etc.

6.11



Graph: $3x < 7$ on the number line.

6.12



Graph: (-3,2), (+3,-3), (0,-1)

6.13 Using the coordinate axis of 6.12, graph the solution for $\{(x,y): 2x + 3y > 10\}$

6.21 Construct a frequency table for this set of unorganized data:
(Scores by female college students on the MNA)
4,14,25,3,7,16,28,17,13,14,2,3,4,
19,18, 21,22,16,15,17,16,19,18,
27,26,25,24,23,19,18,17,16,5,4,3

6.221 Construct a picture graph for the following data:
1950- 12 boys, 15 girls
1955- 16 boys, 19 girls
1960- 21 boys, 17 girls
1965- 25 boys, 21 girls
Students attending Lincoln Jr. High School, Mrs. Tops class.

6.222 Interpretation

6.23 Measures of central tendency
6.231 Mean

6.232 Mode

6.233 Median

6.24 Measures of Variability

7. Probability
7.1 Chance Events

7.2 Key concepts
7.21 Experiments

7.22 Sample Space

6.231 $\bar{X} = \frac{\sum X}{n}$

6.232 The mode is the most frequently occuring score in a set of scores.

6.233 $Mdn = \ell + \left(\frac{\frac{n}{2} - \sum f_b}{f_w}\right) i$

6.24 Measures such as the standard deviation, variance, and interquartile range are examples of measures of variability.

$$s^2 = \frac{\sum X^2 - \frac{(\sum X)^2}{n}}{n-1}$$

7.21 An experiment is an operation or procedure for discovering something unknown, for example the tossing of a coin 50 times to see how many heads come up.

7.22 The set of all possible outcomes of an experiment is the sample space.

6.222 On the
How much mo
recreation
earns $9,00

How a typi
the money

6.231 Calcul
scores f

6.232 Calcul
scores f

6.233 Calcul
scores f

6.24 Calcula
deviatio
6.21

7.1 What are
will rai

7.21 Toss a
record the

7.22 The sam
a die is

S =

Interpretation

es of central tendency
Mean

Mode

Median

es of Variability

ts

s
ments

Space

6.222 On the circle graph below, How much money is spent on recreation by a family that earns $9,000 per year?

How a typical family spends the money it earns(in %).

6.231 $\overline{X} = \frac{\sum X}{n}$

6.232 The mode is the most frequently occuring score in a set of scores.

6.233 $Mdn. = \ell + \left(\frac{\frac{n}{2} - \sum f_b}{f_w}\right) i$

6.24 Measures such as the standard deviation, variance, and interquartile range are examples of measures of variability.

$$s^2 = \frac{\sum X^2 - \frac{(\sum X)^2}{n}}{n-1}$$

7.21 An experiment is an operation or procedure for discovering something unknown, for example the tossing of a coin 50 times to see how many heads come up.

7.22 The set of all possible outcomes of an experiment is the sample space.

6.231 Calculate the mean of the scores from 6.21

6.232 Calculate the mode of the scores from 6.21

6.233 Calculate the median of the scores from 6.21

6.24 Calculate the standard deviation for the scores of 6.21

7.1 What are the chances that it will rain today?

7.21 Toss a die 50 times and record the results of each toss.

7.22 The sample space for tossing a die is

$$S = \left\{ \cdot, \cdot\cdot, \therefore, \cdot\!:\!\cdot, \cdot\!\cdot\!\cdot, \cdot\!\cdot\!\cdot\!\cdot \right\}$$

7.3 The probability of an outcome as a number between 0 and 1.	7.3 By definition, $P(E_o) = 1$, where E_o is an event certain to happen, $P(E_n) = 0$, where E_n is an event that cannot occur.	7.3 What is tossing a toss of a
7.4 Addition and multiplication in probability	7.4 If two or more events are independent, the probability that they will all occur is the product of their separate probabilities. The probability that any one of a number of mutually exclusive events will occur is equal to the sum of the probabilities of the separate events. Thus, $P(A \wedge B) = P(A) \times P(B)$ and $P(B \vee C) = P(B) + P(C)$	7.4 What is tossing a a two foll that gives What is you will a a (4,6) o two dice?

* * * \

y of an outcome as a 0 and 1.	7.3 By definition, $P(E_o) = 1$, where E_o is an event certain to happen, $P(E_n) = 0$, where E_n is an event that cannot occur.	7.3 What is the probability of tossing a 1 (·) on the first toss of a die? Ans. 1/6
ultiplication in	7.4 If two or more events are independent, the probability that they will all occur is the product of their separate probabilities. The probability that any one of a number of mutually exclusive events will occur is equal to the sum of the probabilities of the separate events. Thus, $P(A \wedge B) = P(A) \times P(B)$ and $P(B \vee C) = P(B) + P(C)$	7.4 What is the probability of tossing a die and getting a two followed by a second toss that gives you a six? What is the probability that you will get either a (2,3) or a (4,6) on a single toss of two dice?

* *

t would be appropriate to extend this Guide to include a multiplicity of content items sed to correlate mathematics with the other subject matter disciplines. There is little whether, but when and what! However, this will not be done for several reasons, not ich is this author's ignorance of the other disciplines. Suffice it to say, as did the ue of Phi Delta Kappan, that total integration of the disciplines is nearer and more further development of content and pedagogy for any single subject matter discipline. d Weingartner's book Teaching as a Subversive Activity by Delacorte Press for use of the h as a real alternative to the disciplines approach. You'll find the reading well worth

has been done in this area, however, students will need to depend on local teachers and ittees to develop the inquiry approach to a totally integrated approach to learning. Or, not do it, it may be done for them.

SOME PRACTICAL ASPECTS OF INDIVIDUALIZED INSTRUCTION

Now that curriculum and theoretical matters have been examined for the individualization of mathematics instruction, it becomes important to discuss implementation of individualized programs of instruction. It is only as teachers can tackle the problems of executing a program of individualized instruction that they can succeed in this monumental task. Much has been written elsewhere about such matters as team teaching, departmentalized approaches, etc. However, we will deal with items which can be used in an individualized program regardless of combinations of teachers, students, and administrators. Of concern here are the following: Student Learning Guides, Student-Learning-Guide Coding Procedure, Student-Learning -Guide Achievement Record, Student-Learning-Guide Record Form, Student Daily/Weekly Planning Schedule, and that perennial problem-maker, Evaluation of Pupil Progress and Reporting to Parents. Other matters which could be discussed here are best left to the individual teachers or teams. The guides and forms that are presented are suggestive, and, as individual teachers or districts prefer modifications for their unique set of circumstances, these modifications should be made.

STUDENT LEARNING GUIDE (SLG)

The Student Learning Guide is a modification of Esbensen's Contract. It consists of the SLG code, the purpose of the objective, the behavioral objective, a sample evaluation consistent with the objective, and the available learning activities to help the student achieve the objective. Esbensen's contract form was modified because of experience this writer has had with students in using contracts. No attention was given to the Content Classification and the Taxonomy Category elements of the contract by students. These two elements have been incorporated into the code for the SLG, thus retaining their function for the teacher or curriculum committee.

Each SLG requires an evaluation to determine whether the student has achieved the desired behavior. The evaluation must be consistent with the SLG Desired Behavior. In many ways, the SLG Desired Behavior and the Evaluation for the SLG can be thought of as equivalent---different forms of the same thing.

(Figure 1 and Figure 2 are offered as examples of an SLG and Evaluation form.)

4.2-2181
C-2

Student_____
Due date_____

Why? (Purpose)

To develop skill in generating the set of factors of a number in preparation for finding the greatest common factor of two numbers.

What? (Desired Behavior)

Given ten numerals that name whole numbers not greater than 81, the student is able to list the set of factors of each number with 90% accuracy.

(Sample Evaluation)

List the set of factors of each number.

1.54 2. 72 3. 12

How? (Learning Activities)

___A. Mathematics for the Elementary School, Book 5, pp. 53-55.
___B. Elementary School Mathematics, Book 5, pp. 170-173.
___C. Elementary School Mathematics, Book 6, pp. 88-91.
___D. Elementary Mathematics-Concepts, Properties, and Operations, Book 5, pp. 72-73.
___E. Cyclo-teacher M-79, M-79a, M-80, M-80a.
___F. Drilltapes, Enrichment Topics, Reel 1.
___G. Filmstrip 400534, "Factors, and Primes".
___H. Small-group, teacher-led.
___I. Small-group, student-led.

(Figure 1: Sample Student Learning Guide)

4.2-2181
C-2

Evaluation

List the set of factors of each number.

1. 48
2. 24
3. 81
4. 12
5. 36
6. 54
7. 72
8. 16
9. 42
10. 68

(Figure 2: Sample Evaluation)

STUDENT LEARNING GUIDE ACHIEVEMENT RECORD

The SLG Achievement Record was devised to supply the teacher with two types of information: which students are on the same SLG? and on what day is each SLG due? The form also provides a record of each student's achievement in a given area.

Because the teacher can readily observe the number of students on a given SLG, he may more easily schedule small group presentations. A quick check on the due dates will also reveal which students have SLG's due. He may thereby assist students having difficulties completing specific objectives. Even though the goal of individualized instruction is complete self-direction by the student of his own learning, realistically one must admit that different students achieve this goal at different times and to different degrees. Therefore, the teacher must offer guidance to those who need direction until the self-directed student emerges.

Figure 3 is an example of an Achievement Record. The names are fictitious to protect the innocent. Space limitations prevent the completeness of the Record for the entire set of students involved. Naturally, in classroom use, the SLG numbers, due dates, and number of student names would each be adjusted

to fit the unique circumstances of the particular teacher or team of teachers.

Though the student has a number of objectives (SLG's) to complete for a given period of time, he generally works on one SLG at a time. Thus, in using the Achievement Record, the teacher simply draws a diagonal for the appropriate SLG number and record s the SLG number under the agreed to due date. On successful completion of the SLG, the teacher completes the other diagonal of the rectangle and places a (+) next to the objective number under the due date. For quick observation, the teacher might also pencil in the rectangular region associated with a completed SLG number. It will also be noted that the students have been partitioned into groups of four students each--again for quick reference. Experience has shown that unless the students are thus divided, teachers' eyes begin to play tricks on them as they record information.

Short of computer help, which often turns out to be more work for the teacher, this record form has proven to be most helpful. A single record sheet for each student requires much extra effort to achieve groupings of students and to determine when individual students have specified SLG's due. The purpose of this form, then, is obvious: to make information available to the teacher as

quickly and as conveniently as possible so that she may spend her time helping students rather than keeping records.

SLG ACHIEVEMENT RECORD-MATH (2000)

SLG Number							Student Name	Due Date				
541	009	035	106	670	132	432		3/4	3/5	3/8	3/9	3/10
		X				/	Abrams, B.	35+				432
X				/			Akins, T.	541+			670	
							Blackwood, G.	9+				
/	X		X		X		Chamblis, S.	9+		106+	132+	
				/			Dangerfield, F.			670		
X		X	/				Eoff, W.		35+		541+	
					X	/	Feezel, R.	132+			432	
							Goodpasture, J.			9		
			X	/		X	Heffner, H.	106+		432+		
				/			Jett, G.			670		
	/				X		Johnson, V.	132+			9	
			X			/	Kendall, G.	106+				432
/	X						Koppersmith, F.		9+			
				/			Luther, M.			670		
	/						Ogletree, W.				9	
		X			/		Pancake, C.	35+		132		
	X	X	X	X	X	/	Ragsdale, R.	9+	35+	106+	670+	132+
/							Rickles, D.			541		
	/			X			Rogers, R.	670+			9	
			X		/		Ryman, P.		106+			132
		/					Schorseen, D.			35		
/			X				Sumption, W.	106+			541	
				/			Taaffe, T.		670			
	X					/	Youngblood, E.		9+			432

(Figure 3 - A Sample SLG Achievement Record)

SLG ACHIEVEMENT RECORD-MATH (2000)

SLG Number				Student Name	Due Date						
106	670	132	432		3/4	3/5	3/8	3/9	3/10	3/11	3/12
			/	Abrams, B.	35+				432		
	/			Akins, T.	541+			670			
				Blackwood, G.	9+						
X		X		Chamblis, S.	9+		106+	132+			541
	/			Dangerfield, F.			670				
/				Eoff, W.		35+		541+		106	
		X	/	Feezel, R.	132+			432			
				Goodpasture, J.			9				
X	/		X	Heffner, H.	106+		432+			670	
	/			Jett, G.			670				
		X		Johnson, V.	132+			9			
X			/	Kendall, G.	106+				432		
				Koppersmith, F.		9+				541	
	/			Luther, M.			670				
				Ogletree, W.				9			
		/		Pancake, C.	35+		132				
X	X	X	/	Ragsdale, R.	9+	35+	106+	670+	132+		432
				Rickles, D.			541				
	X			Rogers, R.	670+			9			
X		/		Ryman, P.		106+			132		
				Schorseen, D.			35				
X				Sumption, W.	106+			541			
	/			Taaffe, T.		670					
			/	Youngblood, E.		9+			432		

re 3 - A Sample SLG Achievement Record)

STUDENT LEARNING GUIDE CODING PROCEDURE

Seven areas are traditionally defined for curricular development in the elementary school. In addition, an eighth area, Personal/Social Development, should be added to include much needed individual development in the Affective Domain. If one considers the area of the Humanities to be separate, there would be nine. However, the Humanities program more logically is identified as an element of unity among the other eight areas. To view it as a separate subject is to further fragment instruction, which is not, by definition, the purpose of the Humanities.

The eight subject areas are:

1.	Language Arts:	La-series 1000
2.	Mathematics:	Ma-series 2000
3.	Science:	Sc-series 3000
4.	Art:	Ar-series 4000
5.	Music:	Mu-series 5000
6.	Social Studies:	So-series 6000
7.	Physical Education:	PE-series 7000
8.	Personal/Social Development:	PS-series 8000

The series number that appearswith each curriculum area simply indicates, for bookkeeping purposes, the numbers available within the subject matter area for SLG identification. For example, Social Studies may use the series of numbers

that fall between 6000 and 7000. This provides the range of numbers needed to designate the many SLG's one would provide for any program of individualized instruction. This range was chosen because a "multiple of 10" series would provide 10 objectives for a given area, a "multiple of 100" series would provide 100 objectives, whereas the "multiple of 1000" series we are using provides 1000 objectives in any given area. From experience with individualized instruction it can be shown that while one will have more than 100 objectives in a given subject area, it is doubtful that one will have more than 1000. In fact, 1000 objectives will probably cover the range of objectives for a given area for the entire elementary grades.

The coding procedure for each SLG would follow this format:

	Example
Subject area/content outline/series objective number	Ma3.1412-2076
Taxonomy domain/category	C-5

In our example, the Ma (subject area) is self-explanatory.

The 3.1412 ties the What? (Desired Behavior) of the SLG to the Mathematics Content Guide, a portion of which appears below.

3.14 Numeration Systems
- 3.141 Base-ten numeration
  - 3.1411 Expanded Notation
  - 3.1412 Reading and Writing names for numbers

3.1413 Rounding numbers and Estimation
3.1414 Exponential Notation
3.1415 Scientific Notation

The outlining procedure suggested for the outline of content is the decimal system of subdivision. It provides the needed flexibility for outline format that cannot be achieved with the alternating letter and numeral format. Also, in terms of the coding procedure, the main subdivisions of this outline (3.) are associated with each subheading (3.1412), thus making communication more simple. This format should most easily lend itself to computerization.

After the dash in the first line of the code comes the Numeral Designation for the SLG within the given subject matter area. No two SLG's throughout the entire curriculum will have the same series number, thereby avoiding possible communications mix-ups.

Finally, listings are needed of the Taxonomy Domain and the Category for each SLG. In our example, we note the objective to be in the Cognitive Domain, the 5th Category--Synthesis. The domains and their categories are listed here for reference purposes.

1. Bloom's Taxonomy - Cognitive Domain
   1.1 Knowledge
   1.2 Comprehension
   1.3 Application

1.4 Analysis
1.5 Synthesis
1.6 Evaluation

2. Popham's Taxonomy - Cognitive Domain
   2.1 Lowest
   2.2 Higher than the lowest

3. Esbensen's Taxonomy - Cognitive Domain
   3.1 Knowledge
   3.2 Comprehension
   3.3 Application
   3.4 Invention

4. Krathwohl's Taxonomy - Affective Domain
   4.1 Receiving (Attending)
   4.2 Responding
   4.3 Valuing
   4.4 Organization
   4.5 Characterization by a Value or Value Complex

5. Simpson's Taxonomy - Psychomotor Domain
   5.1 Perception
   5.2 Set
   5.3 Guided Response
   5.4 Mechanism
   5.5 Complex Overt Response

Many educators feel Bloom's Taxonomy to be unduly complicated making the assignment of a given objective to the appropriate category within the taxonomy more difficult than it needs to be. At the other end of the taxonomy scale we have Popham's Taxonomy, consisting of only two categories. This Taxonomy

classification scheme is too simple, not taking into account the fact that a binary classification can hardly do justice to the vast range of educational objectives with which students need to become involved in the course of their educational experiences in the schools. A "middle-of-the-road" scheme that might suit a classroom teacher's use is Esbensen's Taxonomy. Consisting of four categories it makes the job of classifying cognitive objectives a bit simpler than Bloom's, but not so simple as to make the use of a taxonomy seem unimportant.

The affective domain represents a domain that traditionally has been left to chance--much talk but little action. It should be noted that work is beginning on many fronts in this domain, but the teacher should not hold his breath waiting for the results.

The coding of objectives, SLG's, is important because it is only be developing a coding scheme that one can seriously undertake the sometimes awesom task of recording pupil progress in an individualized program of instruction. A systematic coding procedure also allows teachers a way of keeping track and finding objectives that have been developed.

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STUDENT DAILY/WEEKLY PLANNING SCHEDULE

The student, under teacher guidance, should plan his Daily/Weekly schedule. In reality, the student will probably end up planning only a portion of the week at the beginning of the week, followed by daily updatings and modifications. He records the work which he will be doing, depending on small group and other sessions scheduled for the day. By planning his own schedule the student becomes more aware of the importance of utilizing his time efficiently. His own responsibility for completion of his work is thereby increased. Although initially some students need much teacher help, they must be weaned to assume more and more of the responsibility themselves. Part of "learning how to learn" is "learning how to plan".

The teacher should not demand close adherence to the schedule once completed. Rather flexibility in planning should be the goal. For example, a student may schedule a science experiment only to find the equipment unavailable. Resolution of the situation is made with the selection of an alternative activity in science or in another area.

Figure 4 is an example of one child's schedule. The specific objective that the student is working on does not need to be listed. The subject area should s

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Figure 4

STUDENT DAILY/WEEKLY PLANNING SCHEDULE

	Monday (4/6)	Tuesday (4/7)	Wednesday (4/8)	Thursday (4/9)	Fri
9:00-9:30	*Homeroom & Planning*	*Homeroom & Planning*	*Homeroom & Planning*	*Homeroom & Planning*	*Ass*
9:30-10:00	*L. A.*	*L. A.*	*L. A.*	*L. A.*	*Ass*
10:00-10:30	*Math*	*L. A. Test (Spelling)*	*L. A.*	*L. A. (Test)*	*L.*
10:30-11:00	*P. E.*	*P. E.*	*P. E.*	*P. E.*	*P.*
11:00-11:30	*Math*		*Music*		*Mus*
11:30-12:00	*S. S.*	*Math*	*Math (Test)*	*Math*	*Ma*
12:30-1:00		*Sci.*	*Math*	*S. S.*	*Ma*
1:00-1:30	*S. S. (Test)*	*Sci.*	*Art*	*S. S.*	
1:30-2:00	*Sci.*	*Sci.*	*Small Group Discussion*	*S. S.*	
2:00-2:30	*Art*		*How we treat other*	*S. S. Project*	*Ar*
2:30-3:00	*Sci.*	*Music*	*people*	*Outside Speaker on*	
3:00-3:30				*Race*	

STUDENT DAILY/WEEKLY PLANNING SCHEDULE

/6)	Tuesday (4/7)	Wednesday (4/8)	Thursday (4/9)	Friday (4/10)
&	*Homeroom & Planning*	*Homeroom & Planning*	*Homeroom & Planning*	*Assembly*
	L. A.	*L. A.*	*L. A.*	*Assembly*
	L. A. Test (Spelling)	*L. A.*	*L. A. (Test)*	*L. A.*
	P. E.	*P. E.*	*P. E.*	*P. E.*
		Music		*Music*
	Math	*Math (Test)*	*Math*	*Math Project*
	Sci.	*Math*	*S. S.*	*Math Project*
	Sci.	*Art*	*S. S.*	
	Sci.	*Small Group Discussion*	*S. S.*	
		How we treat other	*S. S. Project*	*Art*
	Music	*people*	*Outside Speaker on*	
			Race	

STUDENT SLG RECORD FORM

The Student SLG Record Form consists of a listing of objectives or SLG's in each area of the curriculum for the individual student. This listing is made after appropriate diagnostic procedures have been completed. It is completed by student and teacher is concert. The list is made for an appropriate length of time, probably the traditional marking period, although there is nothing sacred about this time period. (note the discussion considering Pupil Evaluation and Reporting to Parents.) One will note that spaces have been provided to allow student choice of objectives (SLG's) or for needed addition of objectives by student and teacher during the period. This, then, is not an inflexible record form but rather suggestive in its intent. It gives both teacher and student some tentative goals for which to aim during the period. This matter of setting goals is one of the weaknesses of most individualized instructional programs.

The student has a record for all of his work for the period and can note his progress toward the goal of completion of this work determined by both himself and his teacher. This listing is <u>minimal</u>, however, and is not meant to

establish a final listing of work for the entire evaluation period. It is felt that student and teacher should decide on minimums to be completed for the period, but once these SLG's have been completed, additional goals for the period should be set by the student.

Since a student generally works on one SLG per area at a time, this record will be his constant guide. He lists due dates, determined by himself and the teacher, directly beneath the appropriate SLG number and marks out those objectives that have been completed. This record of one page includes every subject area and is more than just a guide to the student. Along with copies of the SLG's he is currently working on, this record form becomes a guide to parents as to their own son's or daughter's progress during the period.

Obviously, the SLG numerals will be different from subject area to subject area for different students, since students vary in their achievement and since it is impossible to equate two objectives in two different areas. Even though each subject area has a "multiple of 1000" series associated with it, three-digit numerals are printed on this form. The printing of the fourth digit, 2 in the case of mathematics, would be a waste of paper and space, providing no useful information when printed over and over again.

Figure 5 consists of the record of one student for one marking period. The PS series is established through a diagnostic procedure, as would the other areas. However, further SLG's for this particular student will not be necessary unless some particular problem arises during the marking period.

STUDENT SLG RECORD FORM

Name_____
Comp. Date_____

La-1000 series													
007	008	009	013	019	054	055	056	057	076	077	078	079	080
085	089	___	___	___	___								
Ma-2000 series													
141	142	143	144	145	146	147	148	150	155	160	161	162	___
___													
Sc-3000 series													
234	235	236	240	300	301	302	303	341	___	___	___	___	___
___	___												
Ar-4000 series													
189	190	191	201	231	___	___	___	___	___	___	___	___	___
Mu-5000 series													
222	223	224	225	226	227	228	241	243	249	250	261	278	
So-6000 series													
467	468	469	470	472	472	489	491	500	543	544	545	___	___
___	___	___											
PE-7000 series													
456	___	___	___	___	___	___	___	___	___				
PS-8000 series													
011	031	037	041										

Figure 5 - Sample Record Form

FACILITIES FOR INDIVIDUALIZED INSTRUCTION

Innovative programs of individualized instruction call for facilities that differ markedly from the traditional school design of equal-sized boxes called "classrooms". Though the traditional design may be administratively convenient, it is doubtful whether this design promotes individual student learning. In the construction of new buildings for individualized programs the key word has been that of "flexibility". In fact, one of the significant high school innovations has been the "flexible modular schedule". Flexibility, however, is a highly abstract concept. For educators to tackle the design of an appropriate facility for their innovative programs, this term must be broken down. ARchitect William Caudill has abandoned flexibility for more specific terms:

1. Expandible space - allowing for ordered growth
2. Convertible space - adapting to program changes
3. Versatile space - serving many functions
4. Malleable space - changing space at once and at will

With building costs soaring ever higher, it is not always feasible nor desirable for a school district to build new facilities for its innovative programs. It is, however, important to consider that the learning environment be arranged so that learning is facilitated, not retarded. This author has seen

school buildings that could not possibly facilitate a traditional program, let alone an innovative one. One must be careful not to place an innovative program in a building that puts an undo burden on student learning.

How then should the learning environment, new or old, be arranged for maximum learning? Provision must be made for elements of the following in any individualized program:

1. Independent study - individual student investigation in a setting where distractions are kept at a minimum. In the majority of cases this should be the instructional materials center.

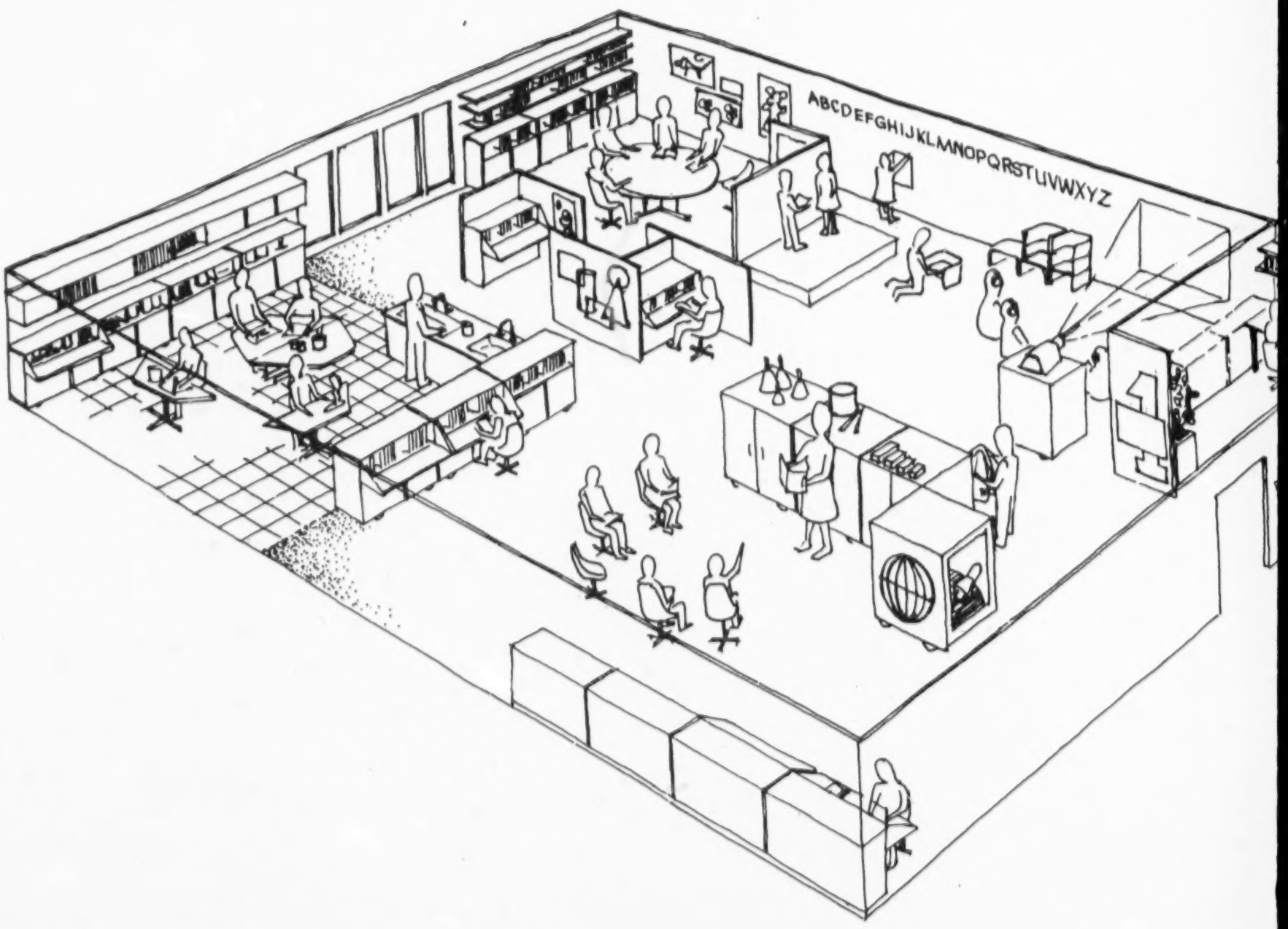
2. Dialogue - one student meeting, formally or informally, with one or more teachers in a semi-private environment.

3. Small-group discussion - a group of less than 10 students working with (tutorial) or without (interaction) a teacher.

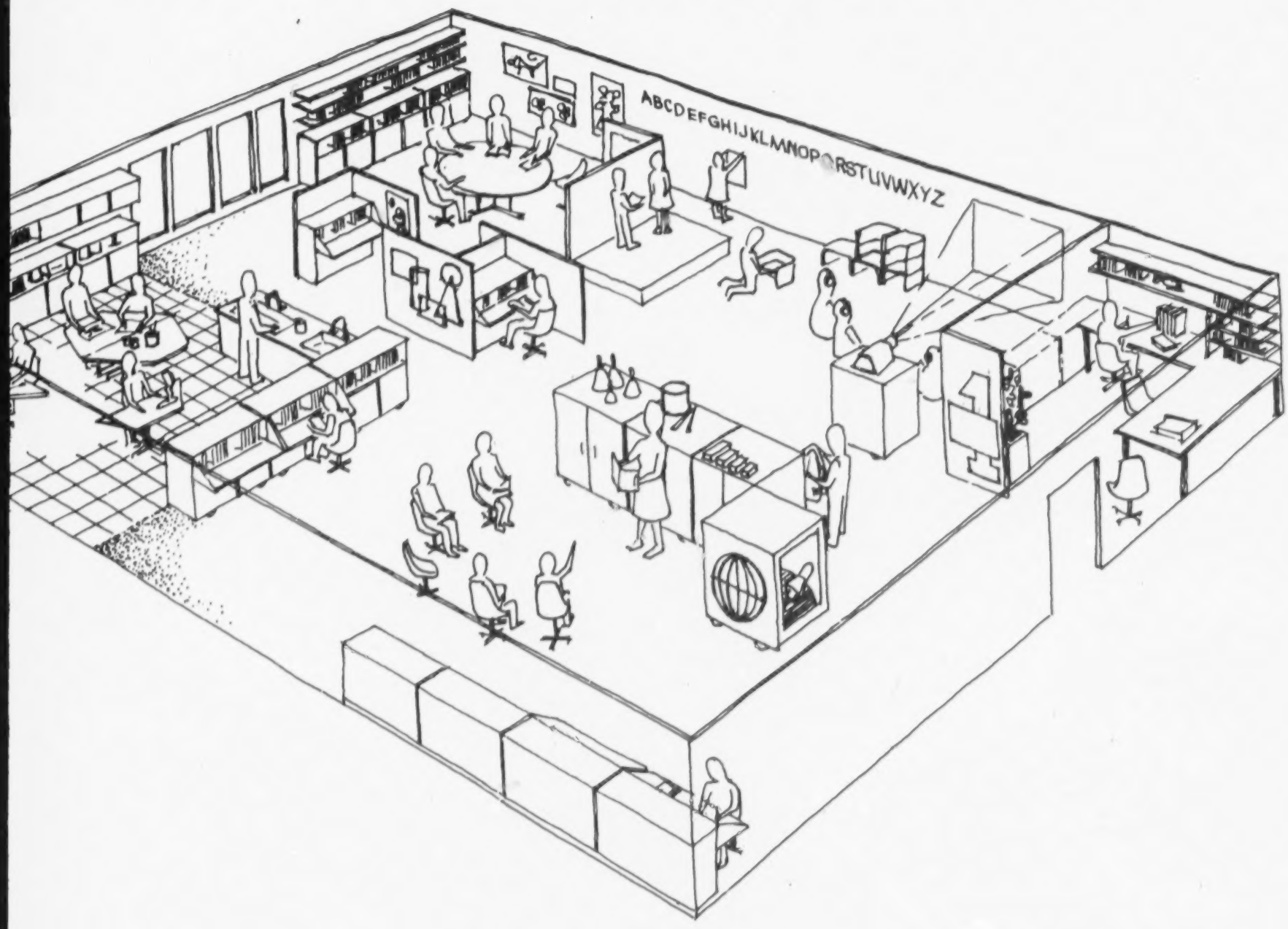
4. Large group presentation - groups of 15 or more students meeting with one-way communication emanating from the teacher. (This method should be used sparingly.)

When discussing individualized instruction with teachers, one question that invariably comes up is: How do I arrange my own classroom to provide for individualized instruction? The answer to this question can be visualized best by utilizing two illustrations: one a classroom converted into a room for individualized instruction in the primary grades (Drawing A), and the other, an

open design for intermediate grades (Drawing B). Both drawings are originals of the Industrial Designer, R. Edwin Wilgus.

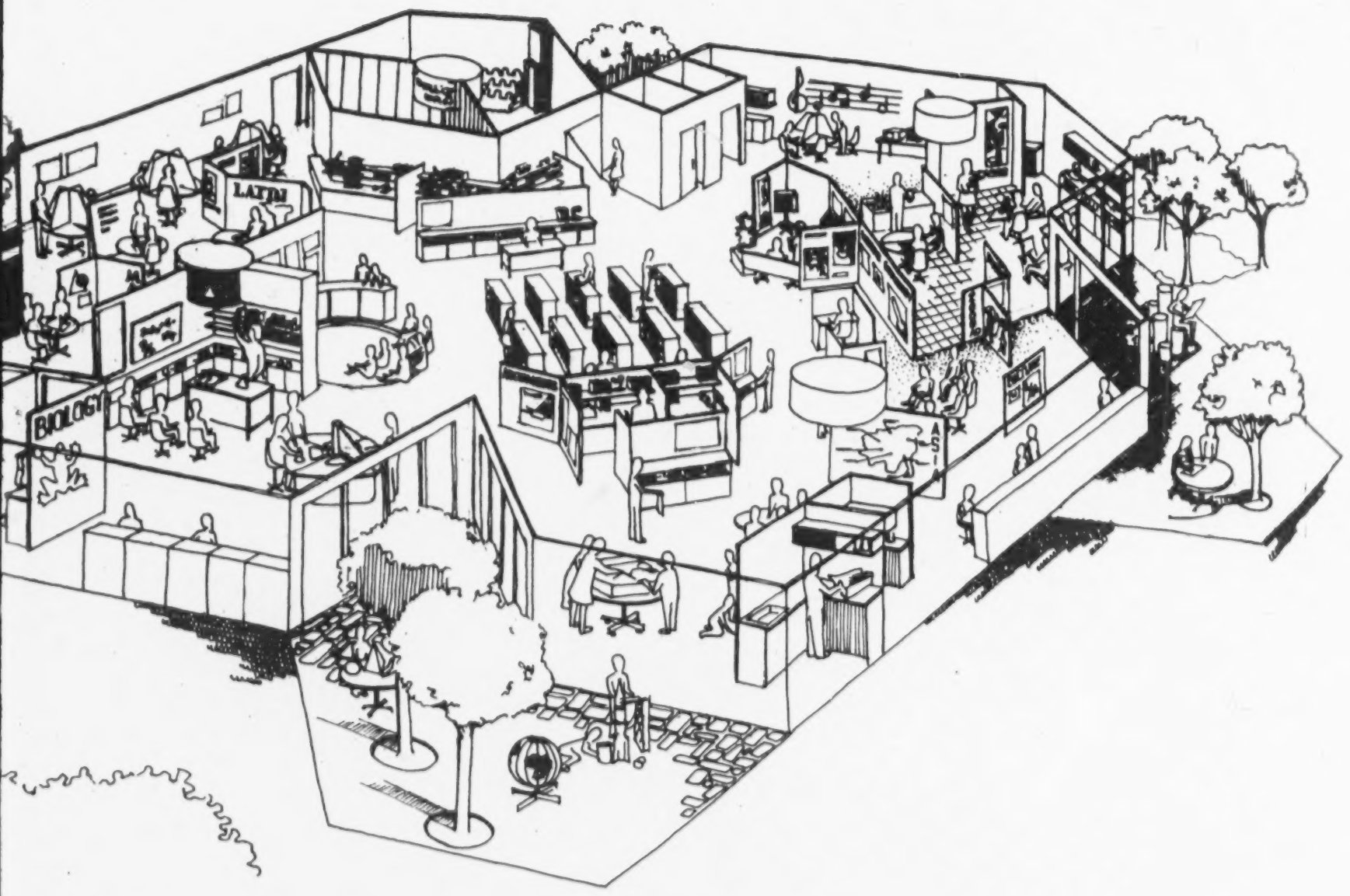


DRAWING A – CONVERTED CLASSROOM FOR INDIVIDUALIZED INSTRUCTION



DRAWING A – CONVERTED CLASSROOM FOR INDIVIDUALIZED INSTRUCTION

DRAWING B-OPEN DESIGN FOR INTERMEDIATE GRADES



DRAWING B-OPEN DESIGN FOR INTERMEDIATE GRADES

The detailed nature of these drawings answers many of the questions about the arrangement of a classroom for individualized instruction. However, certain elements which are essential to the learning environment for individualized instruction should be noted. As one considers both primary and intermediate grades, these elements are more similar than different.

The primary classroom provides traditional staffing and student patterns: 1 teacher for 30 students. Its area of about 900 sq. ft. has been converted from a traditional classroom into a modern, open classroom suitable for the application of contemporary learning theory in early childhood education. The intermediate classroom was designed specifically for a differentiated staff working with 100 to 120 students in an open environment of about 4000 sq. ft. (excluding the outdoor patio areas). The essential features common to both learning facilities are:

Teacher/team planning areas,
Audio-visual equipment and materials for student use,
Extensive storage space in the form of movable and permanent cabinets and shelving,
Almost exclusive use of tables and chairs, except for the individual study desk,
Display areas for student use,
Both individual, small group, and large group areas,
A specific area for the dramatic and creative activities of students,

Acoustical ceilings and carpeted floors except for art and project areas.

The primary classroom space is generally convertible versatile, and malleable. This offers the teacher many options in the arrangement of the open classroom area. When such a classroom is set up, two major questions need to be considered:

1. What traffic patterns will develop as a result of material and furniture placement?

. Will these traffic patterns assist student learning or will they result in wasted time and disruption?

Obviously, if all materials for reading, math, art, and music are in one corner of the room, difficulties will arise since students will cluster in that area. This in turn causes behavior problems and noise levels to rise. Thus, a teacher must place materials and arrange areas so as to facilitate free movement and limit "congregating".

2. Therefore, are the materials that I expect students to get for themselves readily accessible? In an individualized classroom a teacher has enough to do without filling student requests for such things as manilla paper or water-colors. A little forethought here will be a great time saver in this regard.

These two questions regarding materials, furniture, and their arrangement, apply even moreso to the intermediate area of Drawing B. Though the number

of staff members and the amount of space are greater, so is the number of students in the area. In the intermediate design, specific areas have been noted for student work and materials placement. Beginning at the very top of Drawing B and proceeding clockwise we have:

1. Auditorium - capable of being divided into 3 separate sound-proof areas for the creative and dramatic arts including music. Note the unique A-V facilities used here and throughout the classroom.

2. Music area - with two acoustically treated practice rooms and special audio carrels.

3. Art area - non-carpeted with sinks, tables, special drawing tables and chairs, and nearby "mud room".

4. Humanities area - seminar area for discussion purposes.

5. Social Studies area - special projects and preparation rooms.

6. Science area - with special tables, study areas and demonstration center. Note the animal room and special projects area.

7. Mathematics area - note the provision for manipulative devices and calculators.

8. Language Arts - including special audio carrels.

9. Materials Center - central storage for print and non-print materials. Note the recessed group area and the current events lounge with television. On either end of the center is to be found the four teacher/team planning areas.

10. Patios - used as expandible space for student need in any area.

One of the main reasons for the identification of specific subject matter areas is the unique requirements that several areas place on equipment, furniture, and facilities. Science and music are prime examples. However, except for these cases, all of the other spaces are defined as expandible, convertible, versatile, and malleable. Teachers in individualized programs have found that space can be used in many varied and unforeseen ways.

Teachers need to develop an awareness of how facilities may add immeasurably to the success of student learning. Teachers should also remember that changes can be made in furniture and materials arrangements as the year progresses if the existing patterns do not seem to function well. Students enjoy change. It gives a new perspective to what appeared a closed situation.

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EVALUATION OF PUPIL PROGRESS

One of the persistent problems in education is the evaluation of individual student progress in the program of instruction. Hundreds of committees composed of hundreds of teachers have written thousands of pages about methods of reporting procedures to be used in delineating to parents the improvement or lack of improvement of students. Teachers spend literally hundreds of hours each year in parent-teacher conferences attempting to convey to oftentimes overwrought parents the weaknesses and successes of pupils. Though much time and effort is expended, we still hear from parents that "these teachers never say anything," and from teachers the familiar "Glad those conferences are over" or "I never really know what to say to some parents. My comments get to be repetitious one conference to the next." What makes communication to parents such a difficult problem? Is it because we lack a philosophical base? Is it because parents want those A, B, C, D, F grades and cannot understand anything else? Or is it a matter of general indifference on the part of both parties? Do programs of individualized instruction help resolve these communication problems or do they tend to intensify the difficulty?

In attempting to offer a solution to this problem of pupil progress, one must begin with a sound rationale of individualized instruction and then make a consistent effort to establish evaluative procedures. To be in accord with our previously stated rationale, one would come to see that the individual student's evaluation of his own progress is vital. In a program that focuses on the learner and the concept of uniqueness of perception, the most important evaluation is precisely the one that the student would make of himself and his progress. Testing procedures, as are known today, would represent isolated bits of information that the student might or might not use toward the eventual successful completion of certain goals. What about the first and second grade students who have not developed the necessary skills prerequisite to good decision-making? This would not mean that a teacher or parent would abandon a young elementary child who lacks confidence, an understanding of individual responsibility, or the ability to make the necessary judgments about his own progress. This self-evaluation program would require more initial guidance on the part of the teacher and parent, but in a different direction than is the case in public education today. Evaluation and testing would be a means of guidance to the child as immediate feedback for future decisions, not an opportunity

for teacher and parent alike to psychologically molest and demean the child. The concept of failure would not exist as part of evaluation. The natural outgrowth of this sort of evaluation procedure would be that the parent and teacher would become, in a sense, unnecessary. Students involved in such a program would, by the teenage years, be able to make responsible choices based on whatever information was made available to them by their immediate environment. The real difficulty in our present educational program is that we expect students to make responsible choices--ones with which we agree-but ones that have never been encouraged within students because the development of the skills that are prerequisite to such responsible decision-making has been neglected.

With the concept of student self-evaluation at the heart of pupil progress in an individualized program, how might a school district or system implement such an evaluation scheme? What would such a program look like? To answer these questions we must walk a diversionary path momentarily in consideration of what might appear unrelated. However, as specifics are discussed the relationship should become clear. In most individualized programs the teachers and administrators are faced with the question: "What happens if a child does

not finish 6th grade work before he goes to junior high school?" In substance at least, the answer invariably boils down to: "We don't know what sixth grade work is and furthermore we take a student as far as he can go while we have him. Then it's up to the next teacher." Indeed, this is about all that can be said in such a situation. In the self-contained classroom, content might have been "covered", but what exactly was learned? Nobody knows of what an elementary education consists, though many people including textbook authors claim to know. By the same token, what does the high school diploma mean? Does it mean that all who receive it have some minimum competencies? Basically, the high school diploma consists of a group of approved courses whose titles, along with appropriate letter grades, appear on a transcript. It certainly tells no one of the particular competencies that students possess when they obtain that "little piece of paper". And, of course, this point could be extended to that most magnificent of all "pieces of paper", the doctorate degree.

This situation could be resolved through the use of the "minimum goals" concept. The establishment of "minimum" standards for elementary, junior, and senior high school would make some sense out of this chaotic situation. In elementary school, minimal standards, in terms of behavioral expectations, might

include aspects of the following areas of educational objectives:

1. Personal and Social Competencies
2. Rational Approaches to the Development of Moral, Valuative, and Ethical Principles
3. Development of Aesthetic and Creative Competencies
4. Academic Competencies
5. Vocational Competencies

For our purposes, it matters not who or what establishes these minimums, although in reality this obviously would not be the case. The point is that these minimums would be established to give us goals for each appropriate level of schooling. Elementary school reading, writing, and arithmetic, along with personal and social competencies, would constitute the thrust of this minimal set of objectives. It is important at this juncture to note that the word "minimum" here used is defined to mean "exactly the least amount possible or allowable." If it meant anything else, the student would never get to anything but minimums in his formal education, as perhaps is now the case.

In addition to minimum standards for all students in elementary school, the curriculum for each student would include areas of individual interest, special abilities, or needs. This portion of a student's school experiences might appropriately be called the "complement". Mathematically, if we consider

the universe of all objectives that a student is involved with in elementary school, where a subset of that universe consists of the minimum goals for all students, then the "complement" of the minimum goals consists of all remaining objectives such that the intersection of the minimum goals and the complement goals is the empty set. It is also quite easy to see that the universe of goals discussed here is an infinite set, as it must be. The complement would consist of all experiences or learning activities beyond the minimums. There would be no concern for overlap from student to student in the objectives to be realized by any given student. If there were overlap, or an intersection, it would be the result of an expressed desire on the part of the student, not on the part of the teacher or parent. The Venn diagram of Figure 1 pictures the relationship of minimum goals and complement goals.

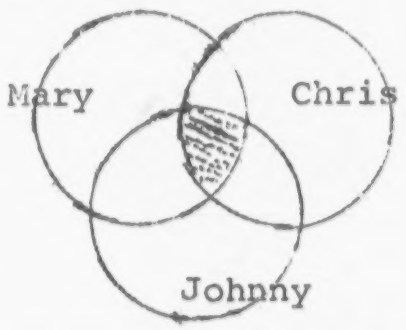

Figure 1 - A Venn Diagram showing a possible intersection of student educational objectives for three students in elementary school. The shaded portion represents common minimum learnings for all.

With the consideration of the totality of a student's school experiences, both minimum and complement, how does one evaluate and report these findings meaningfully to parents and administrators? Student self-evaluation would be a vital first step in the process. A self-evaluation would consist of statements, verbal or written, about successes and problems encountered during a given period of study. The student would discuss help he received, help he would have liked to have received, and what changes need to be made for the next period in order that he may progress in an environment of friendly assistance and eager anticipation. Included in this self-evaluation would be reference to help that parents, teachers, and fellow students might give. Naturally, the self-evaluation must not, under any circumstances, be used in a punitive manner, but in such a way that the student will come to understand himself more fully and come to realize his relation to other people in the school and community in which he lives.

A second step in the evaluation of minimum goals for each student would consist of a visual report including specific objectives completed to date. If

we consider the five areas for educational objectives, the task of reporting minimums would consist of two measures: 1) a cumulative graph showing progress toward ultimate completion of the minimum goals, and 2) a statement of completed objectives to date. The graph, an example of which appears in Figure 2, would be a bar graph constructed so as to provide the area under each bar proportional to the number of objectives in the set of minimums for that area.

Graph of Minimum Expectations

Figure 2 - Mary Jones' graph of accomplishment of minimums for two years of elementary school.

This minimum report of objectives completed is cumulative and can be used for the entire period of time required to achieve the criterion. Both student and parent can easily see how the student is progressing toward the minimum goals. This consideration of minimum goals is only part of the story, however. Both minimums and complements would receive simultaneous attention by the student during any given period. It certainly would be undesirable to have a student work only on minimums until completion of that phase was achieved.

Since the minimums would be established and probably change little in the course of a child's elementary experiences, the CRAM model of achievement monitoring to let the student know how he is progressing toward mastery of the minimum goals appears very desirable.

"CRAM, Comprehensive Random Achievement Monitoring, is a very simple model based on two simple ideas: 1) continuous random sampling, and 2) continuous feedback in terms of final performance criteria." One of the difficulties in any program of individualized instruction is that of testing and keeping records for every objective completed. Instead of testing every objective, students would be tested at random intervals on the goals established as minimums. This way the student would know the objectives he has successfully completed, and which objectives are yet "out there" to be mastered. The CRAM model may need some slight modification for the primary child, but the basic concept would still apply. Since the student would obtain this information of progress at a somewhat regular span of time, this information would also be available to parents and administrators for appropriate assistance at any given point in time. The use of parent-teacher conferences would, in part, center on specific objectives completed and means for further progress toward minimum standards

during the next period of student growth. Since the parents would have at their disposal a set of the minimum objectives, they should be able to be of greater assistance to their children than is possible under the traditional arrangements.

In our program of minimum competencies in elementary school, the emphasis would be on the assistance of the individual student with a continuous flow of data regarding his progress. In turn, the student would be guided to make some responsible choices regarding how he might complete those minimum goals. At all times in reporting pupil progress, the student would be asked for his own personal evaluation, written or verbal, for both teachers and parents so that they might adjust their behavior and assistance to coincide with the needs of the student, not vice-versa. Again, it must be emphasized that the self-evaluations of the student must not be used to chastise, but to aid and assist.

Having stated the society's minimal demands on the student, the exciting portion of the curriculum, the complement, remains for the student to explore, inquire about, and discover the world about him. This complement, which in most cases would constitute the majority of a student's curricular experiences in elementary school, becomes the private domain of the student according to

his interests, abilities, and felt needs. At the elementary level, parents will initially shoulder a portion of those choices, with the aid of the teacher, until the student expresses the desire and ability to make the decisions for himself. For those who know pre-school children, however, it is obvious that they do make their own decisions at a very early age. The major function of the parents and teachers would become that of arranging the environment in school and at home so that a child has many good alternatives from which to choose. Educators and parents have generally done a very poor job arranging an exciting and stimulating environment for children. Much work needs to be done in this area.

An important consideration now becomes the selection of behavioral goals by the student for any selected period of time with respect to the minimum and complement goals. Based on the CRAM feedback of minimums, and other input of parents and teachers including diagnostic testing, the student establishes a goal of numerous specific objectives to be completed over the selected period of time. The student selects his objectives in all the areas of study as a goal to be reached by a certain date. Once that decision is made, it becomes imperative that teachers and parents establish the necessary contingencies to ensure the attainment of the goal.

In evaluation of both minimum and complement goals for a given period of time, the student evaluates himself and his progress. The teacher portion of this evaluation would consist of a statement of completed objectives along with an Efficiency Quotient that may be set for each subject area separately or for the entire set of objectives for the period of time. The parent-student-teacher conference portion of the evaluation would consist of a discussion of the objectives completed, variations between expected objectives to be completed and the actual number mastered, and the necessary input by all for the establishment of goals for the next period of study. This conference would be used for the sole purpose of giving information to the three parties involved to allow for positive pupil growth. The emphasis would be on reinforcement of achievement and self-concept, rather than misbehavior, fear, and failure.

What would the teacher portion of the written evauation look like: First, an appropriate definition of the Efficiency Quotient (EQ) must be given. The EQ is defined as the quotient of the number of objectives completed (observed) to the number of objectives chosen (expected), multiplied by 100. Given this definition, we consider an imaginary John Smith, a nine year old who, with the aid of parents and teachers, set the following goals for a five week period of study:

Subject Area	No. of Objectives
Language ARts	8
Mathematics	10
Social Studies	6
Science	11
Physical Education	3
Art	4
Music	9
Personal/Social	4

Figure 3 - Number of objectives for John Smith for Oct. 1 - Dec. 5. (The specific objectives would be delineated on the Student SLG Record Form.)

Some of the objectives that John has chosen fit into minimum requirements; others are complementary. At the end of the five week period, mastery of chosen objectives follows the pattern in Figure 4.

Subject Area	Expected	Observed	EQ
Language Arts	8	8	100
Mathematics	10	9	90
Social Studies	6	7	117
Science	11	13	118
Physical Education	3	3	100
Art	4	4	100
Music	9	7	78
Personal/Social	4	4	100

Overall EQ = 100

Figure 4 - Report form for a period of five weeks for John Smith concerning mastery of chosen objectives.

As can be observed in Figure 4, John's overall efficiency quotient is 100; yet individual component parts of that quotient are greater or less than 100. What are the reasons for John's failure to achieve his goal in a certain area, and yet "over-achieve" in other areas? Certainly, John could shed some light on this in a parent-student-teacher conference. Hopefully, parent and teacher could also share in the analysis of this situation. Whatever the reasons, we are considering specific information from three sources concerning some very specific goals that teacher, student, and parent knew about and helped to establish. This is a far cry from the statement that one hears in many individualized programs that the parent does not know what is going on any more since everyone is working on different things at different times.

Utilizing this approach to reporting and evaluation, we start off the year with a parent-student-teacher conference to establish initial parameters in a friendly, cordial atmosphere, rather than the usual "wait for six weeks until trouble is a-brewing" to call in parents to obtain valuable information they have regarding their own child. In this program, students would attend elementary school for a minimum of six years; but they might need more time, based on the premise that the learner will succeed in learning a given task to

the extent that he spends the amount of time he needs to learn the task. These statements are general and might be excepted for any particular student.

Nothing has been said about the use of standardized achievement tests and their use in a program of individualized instruction. Little more need be stated now. Most standardized achievement tests fulfill the need for people to partition their fellows into neat compartments that allow any member of that partition to say: "I'm better than you are. Look how much brighter I am than you." If achievement tests can help to improve the quality of the instructional program for the individual child in the form of diagnostic procedures, then they may serve a useful function. Otherwise,...

Thus, an effective program of evaluation for students in an individualized program demands student self-evaluation of his own work. Emphasis is placed on the student-teacher-parent interaction conferences that provide input to the child for the decisions that he must make regarding his school goals. The unique perceptions of each child force on parent and teacher a constant awareneww of their roles from an assistance and information point of view. Use of positive reinforcement techniques, rather than aversive stimulation, allow students to explore their environment in an adaptive, rather than maladaptive,

fashion. The end result, of course, is that a student develop a concept of self that permits him to function happily in whatever situation he should find himself.

References:

DeLay, D. H., and Nyberg, D., "If Your School Stinks, CRAM It," *Phi Delta Kappan*, 51 (February, 1970), 310-312.

COMMERCIAL MATHEMATICS MATERIALS FOR INDIVIDUALIZED INSTRUCTION

This bibliography of materials presented here will be a tremendous aid to the classroom teacher since many, many catalogs of publishers never find their way into teacher's hands for use. The listing of materials in this bibliography was made selectively, not necessarily implying, however, the author's complete familiarity with every item in the list. Most materials of quality appropriate for an individualized program found their way into this bibliography. Text materials are not included for obvious reasons, not the least of which is the fact that textbooks in mathematics at the elementary level have long outlived their usefulness. Likewise, individualized materials such as the IPI materials or the CSMP/CEMREL materials are not included because these materials are not yet ready for general distribution to classroom teachers in the elementary schools of this land.

The bibliography has been divided into nine categories: programmed print materials, manipulative devices*, print materials, filmstrips, games, multi-media kits, transparencies, audio recordings, and calculators. Several resources for

*A more complete bibliography of manipulative devices for general mathematics classes in elementary school is available as follows:
Davidson, Patricia S., "An annotated bibliography of suggested manipulative devices," *The Arithmetic Teacher*, (October, 1968), pp. 509-524.

the teacher have been included as they relate to a specific material and its use with elementary students.

A list of publishers with addresses is given at the end of the bibliography.

PROGRAMMED PRINT MATERIALS

Item No.	Description	Publisher's Code	Price/Unit Item
3-3350	Computational Skills Development Kit	17.(SRA)	$ 68.50
3-3840	Algebra Skills Kit	17.(SRA)	69.95
3-3842	Teacher's Handbook for "Algebra Skills Kit"	17.(SRA)	0.50
50255	Mathematics Learning Center (with Bookcases) Programmed books in every conceivable topic in mathematics - 206 books with teacher's guides.	4.(EBEC)	398.50
TEMAC 50266	Whole Numbers and Numerals, Text and Supplement	4.(EBEC)	8.50
TEMAC 50262	Teacher's Manual (Whole Numbers and Numerals)	4.(EBEC)	1.25
TEMAC 50234	Arithemetic of Whole Numbers, Text and Supplement	4.(EBEC)	8.50
TEMAC 50236	Teacher's Manual (Arithmetic of Whole Numbers)	4.(EBEC)	0.75
TEMAC 59201	Preparing for Algebra	4.(EBEC)	2.75
TEMAC 50298	Modern Algebra, A First Course	4.(EBEC)	12.00
	Exploring Sets, Geometry, and Numeration		
	Program A	7.(HEW)	1.50
	Program B	7.(HEW)	1.50
	Program C	7.(HEW)	1.50

	Individualized Mathematics Drill and Practice Kits		
77031	Kit AA	10.(LWS)	$ 48.00
77041	Kit BB	10.(LWS)	48.00
77051	Kit CC	10.(LWS)	48.00
77061	Kit DD	10.(LWS)	48.00
	Individual Mathematics Program (copyright by Australian Council for Educational Research, 1969.)		
	Set B_1 and Set B_2 (published together)	15.(RL)	60.00
	Set C_1 and C_2 (published together)	15.(RL)	40.00
	Set C_3	15.(RL)	40.00
	Lessons for Self-Instruction in Basic Skills (LSI)		
	Junior Assortment (44 books, 240 student sheets, 1 manual, 4 locator pads)	2.(CTB)	69.00
	Advanced Assortment (44 books, 240 student record sheets, 1 manual, 4 locator pads)	2.(CTB)	69.00
	Contemporary Mathematics (35 books, 96 student record sheets, and 1 manual)	2.(CTB)	56.00
C-Math 403106	Cyclo-teacher Learning Aid 80 Math study wheels	6.(FEP)	14.55
	MANIPULATIVE DEVICES		
M109	Lake and Island Board (linear measurement, area, and volume)	11.(MMD)	9.00

M116	Primary Shapes (set of squares, rectangles, & triangles with 24 graded work cards for use with numbers, measurement, fractions, area and perimeter)	11.(MMD)	$ 10.00
M138	Tangram Puzzles (each tangram packed with its own work card in a poly bag. Includes 2-piece and 3-piece squares, square and 4-triangles, square and 8 pieces, hexagon and 10 triangles, and the Chinese trangram puzzle.)	11.(MMD)	7.00/set
M 160	Madison Project - Independent Exploration Material Contents of kits include: Geoboard and workcards, Tower puzzle and workcards, Centimeter blocks and workcards, Discs and workcards, Weight, springs, and workcards, Peg game and workcards.	11.(MMD)	16.25/set
CX-3E	Cuisenaire Classroom Kit Includes 24 student sets of rods with 72 rods in 10 colors and sizes, *Using the Cuisenaire Rods* (a Photo/text Guide), *Mathematics with Numbers in Color* (book A), *Talks for Primary School Teachers*, Cuisenaire Geoboard and instructional booklets.	3.(CCA)	59.50
CG-5	Filmstrip - "Numbers in Color" A training filmstrip for teachers in the understanding of the Cuisenaire-Gattegno method.	3.(CCA)	5.50
CG-9	Cuisenaire Geoboards A 2-sided plastic board with a 25-peg lattice on one side, and a 17-peg circular lattice on the other.	3.(CCA)	1.20 ea/30 or more
1500	ETA Discovery Blocks A set of 140 blocks in 17 shapes with 15 activity cards graduated from simple to complex to be used with area,	5.(ETA)	12.00/set

	fractions, and spatial relationships.		
18418	Mirror Cards	21.(WDMH)	$ 10.80
18417	Teacher's Guide (Mirror Cards)	21.(WDMH)	2.32
Z80007	Dienes Logical Blocks A set of blocks with games designed to give children experience in attributes and logical operations such as union, disjunction, conjunction, and intersection.	8.(HH)	19.50
Z80008	Dienes Multi-Base Arithmetical Blocks (MAB) Included with the blocks is a set of workcards that, together with the blocks, provide experiences in other number bases beside 10.	8.(HH)	118.00
Z80011	Algebraical Experiences Materials	8.(HH)	59.00
INV 100a	Math balance with 20 weights and See-clips	18.(SEE)	6.75
INV 705	Math balance teacher's guide	18.(SEE)	1.00
PBCKTG	Pattern blocks (ESS) with 3 mirrors and guide	18.(SEE)	12.75
ESA021	Poleidoblocks G with guide	18.(SEE)	20.00
SEE011	Clear Acrylic plastic cubes - 3/4"	18.(SEE)	4.00/100
SEE002	Elementary slide rule	18.(SEE)	1.30 ea./ 10 or more
DIE3/4	Dice, 3/4" jumbo	18.(SEE)	2.50/6 pr.
GN1159	Geometric dominoes	18.(SEE)	1.25
INV202	Polyshpaes (complete material for large polyhedra construction with activity cards)	18.(SEE)	13.00
TAPE04	Measuring tape 30 meters/100 feet	18.(SEE)	7.00
ARN001	Trundle wheel (yard with clicker)	18.(SEE)	4.50
ARN002	Trundle wheel (meter with clicker)	18.(SEE)	5.00
HCP003	Circular protractor	18.(SEE)	0.25
HSC012	Student compass-with unique screw-type pencil/pen holder	18.(SEE)	4.25/doz.

XKIT01	Complete Maths Mini-Lab with equipment, activity cards and guide	18.(SEE)	$ 26.50
XKIT02	Triangle Cards, with equipment and guide	18.(SEE)	9.75

PRINT MATERIALS

M 111	Mathematics Using String by Don Cohen Set of independent exploration cards dealing with distances, thickness, equal lengths, rectangles, index of rotundity, graphing and open sentences.	11.(MMHM)	1.00
M 513	Applied Mathematics Cards Five groups of cards with 30 in each group supplied in 5-ring binders. Each group is complete in itself and each set is developed in increasing order of difficulty.	11.(MMHM)	34.00
M-11	Mathematics Illustrated Dictionary	3.(CCA)	4.50
	Number Stories of Long Ago	13.(NCTM)	2.00
	Polyhedron Models for the Classroom	13.(NCTM)	0.60
	Numbers and Numerals	13.(NCTM)	0.35
	The Number Story	13.(NCTM)	0.85
	Paper Folding for the Math Class	13.(NCTM)	0.60
	Puzzles and Graphs	13.(NCTM)	0.75
3-41	Equations	17.(SRA)	3.60
3-42	Teacher's Manual for "Equations"	17.(SRA)	0.36
	Discovery in Mathematics, by Dr. Davis-Madison Project Teacher's Edition	1.(AWC)	8.00
	Explorations in Mathematics, by Dr. Davis Teacher's Edition (Note: student's editions of the two books by Dr. Davis are also available)	1.(AWC)	8.00

	The Story So Far An outline of, and index to, the ground covered in the Teacher's Guides of the Nuffield Project	22.(JWS)	$ 1.50
	Problems-Green set 52 cards for use with children that present various problems to children. Also contains teacher's guide with answers. (Nuffield Project) Two more sets of cards are in prep=aration.	22.(JWS)	1.67
	Weaving Guides		
	a. Desk Calculators	22.(JWS)	0.95
	b. How to Build a Pond The story of the amazing amount of math that came out of a gift of ducks given to a school and the problems encountered in building the pond. An example of a good class project.	22.(JWS)	1.75
GN0893	Practical mathematics workcards	18.(SEE)	18.75
2-011 thru 2-081	Continuous Progress Laboratory - Mathematics Each continuous progress lesson card contains a list of behavioral learning objectives and a challenge test. This program uses basic textbooks as the major learning resource although audio tapes come with each lab.	23.(EPC)	98.50/ea. level (8 levels)

FILMSTRIPS

F-7 to F-11	Elementary Mathematics: Patterns and Structure A set of 47 filmstrip covering elementary math.	14.(PSP)	$ 258.50
9010 9020	Primary Mathematics filmstrips, series 1 and 2. Two sets of 11 filmstrips.	4.(EBEC)	66.00

GAMES

M-6	TUF, an exciting equations game for children	3.(CCA)	7.00
SEE007	Chip-trading game Includes 800 chips, 8 colors, 4 playing boards, 1 decimal abacus, and instructions (5 players)	18.(SEE)	24.75
ALP001	Perception games	18.(SEE)	3.50
In1128	Vectors	18.(SEE)	5.00
GN0549	Which is more?	18.(SEE)	2.00
MM0160	Quinto	18.(SEE)	8.00
MMT110	Twixt	18.(SEE)	8.00
	Oh-Wah-Ree	18.(SEE)	8.00
XGAM02	Musimatics (games relating math and music	18.(SEE)	
	a. Game of "Measure"	18.(SEE)	2.50
	b. Game of "SET"	18.(SEE)	3.75

MULTI-MEDIA KITS

	Probability and Statistics Kit	16.(SMC)

TRANSPARENCIES

3-4709	Visual Approach to Mathematics Level 1-7 volumes Level 2-7 volumes Level 3-7 volumes Geometry - 7 volumes Sets and Whole Numbers - 7 volumes Rational Numbers - 7 volumes	17.(SRA)

AUDIO RECORDINGS

21- 10000	Drill-tapes	17.(SRA)
21- 10300	Math-tapes	17.(SRA)
PM123F	The Imperial Primary Math Skills Improvement Program This program includes 40 tapes and 30 pupil booklets.	9.(IPI)
IM456	Intermediate Math Program, by Dr. Lola May (40 tapes)	9.(IPI)
	Mathematics Skill Builders Album I - Basic facts, +(10-0), x(0-18)	7.(HEW)
	Album II- Basic facts, +(11-18), x(20-81)	7.(HEW)

MULTI-MEDIA KITS

Probability and Statistics Kit	16.(SMC)	$ 16.95

TRANSPARENCIES

Visual Approach to Mathematics Level 1-7 volumes Level 2-7 volumes Level 3-7 volumes Geometry - 7 volumes Sets and Whole Numbers - 7 volumes Rational Numbers - 7 volumes	17.(SRA)	990.00 entire set. May be ordered as: 1-2 boxes, 195.00 ea., 3-4 boxes 185.00 ea., 6 or more, 165.00 ea.

AUDIO RECORDINGS

Drill-tapes	17.(SRA)	540.00
Math-tapes	17.(SRA)	420.00
The Imperial Primary Math Skills Improvement Program This program includes 40 tapes and 30 pupil booklets.	9.(IPI)	279.95
Intermediate Math Program, by Dr. Lola May (40 tapes)	9.(IPI)	279.00
Mathematics Skill Builders Album I - Basic facts, +(10-0), x(0-18)	7.(HEW)	30.00
Album II- Basic facts, +(11-18), x(20-81)	7.(HEW)	30.00

	Album III - Special Purpose Drills	7.(HEW)	$ 30.00
	Teacher's Manuals for I, II, and III	7.(HEW)	0.30
Range from R-3001 to R03606	Wollensak Teaching Tapes (mathematics) Includes casette or regular tape, teacher's guide, and prepared student work sheets. There are about 60 packages covering many topics from primary to jr. high.	20.(3-M)	7.95/pkg.
2550	Modern Math Concepts Includes tapes on sets, perimeter, area, volume, equation solving, and algebra.	19.(TU)	164.00
	Primary Math Concepts	19.(TU)	36.50 complete
2501	Geometric Shapes		
2502	After		
2503	Before		
2504	Addition		
2505	Subtraction		
	Intermediate Math Concepts	19.(TU)	36.50 complete
2511	Multiplication		
2512	Decimals		
2513	Division		
2514	Fractions		
2515	Combinations		

CALCULATORS

Model 920	Monroe Calculator, with 0,2,4 decimal place option	12.(MON)	$475/10 or more each

SEE001	SEE-Calculator - (transparent)	1-.(SEE)	$ 1.30/10 or more (each)

Many good materials are produced within the local school district that are applicable to a program of individualized instruction. Add these materials that are now available or that you might make for individual children in your own district.

By way of interest, if a school had all of the materials listed in this bibliography the total cost would be $11,411.50, this includes the 10 calculators, etc., to get the cut in price. Naturally, there are some duplications in the materials listed so that a school may not want all of the items listed.

PUBLISHERS

1. Addison-Wesley Publishing Co., Inc.
   Reading, Massachusetts 01867
2. California Test Bureau
   A Division of McGraw-Hill Book Co.
   Manchester Road, Manchester, Mo. 63011
3. Cuisenaire Company of America, Inc.
   12 Church St., New Rochelle, N. &. 10805
4. Encyclopedia Britannica Educational Corp.
   425 N. Michigan Ave.
   Chicago, Ill. 60611
5. ETA School Materials Division
   159 E. Kinzie St.
   Chicago, Ill. 60610
6. Field Educational Publications, Inc.
   902 S. Westwood Ave.
   Addison, Ill. 60101
7. Harcourt, Brace, and World, Inc.
   757 Third Ave.
   N. Y., N. Y. 10017
8. Herder and Herder, Inc.
   232 Madison Ave.
   N. Y., N. Y. 10016
9. Imperial Productions, Inc.
   Dept. K
   Kankakee, Ill. 60901
10. L. W. Singer Co., Inc.
    A Subsidiary of Random House
    Order Entry Dept.
    Westminster, Maryland 21157
11. Math Media Division
    H and M Associates
    P. O. Box 1107
    Danbury, Conn. 06810

12. Monroe
A Division of Litton Business Systems, Inc.
Orange, New Jersey
13. National Council of Teachers of Mathematics
1201 16th St., N.W.
Washington, D.C. 20036
14. Popular Science Publishing Co., Inc.
Audio-Visual Division
355 Lexington Ave.
N.Y., N.Y. 10017
15. Rigby Limited
Melbourne, Austrailia
16. Science Materials Center, Inc.
59 Fourth Ave., N. Y., N. Y.
17. Science Research Associates, Inc.
259 E. Erie St.
Chicago, Ill. 60611
18. Selective Educational Equipment, Inc.
Three Bridge Street
Newton, Mass. 02195
19. Tapes Unlimited
13113 Puritan Ave.
Detroit, Michigan 48227
20. 3M Company
Mincom Division
St. Paul, Minn. 55101
21. Webster Division
McGraw-Hill Book Company
Manchester Road
Manchester, Missouri 63011
22. Wiley, John, and Sons, Inc.
605 Third Ave.
New York, N. Y. 10016
23. Educational Progress Corp.
8538 E. 41st St.
Tulsa, Okla. 74145

SIGNIFICANT DEVELOPMENTAL MATHEMATICS PROJECTS

Central Midwestern Regional Educational Laboratory-Comprehensive School Mathematics Program, Burt A. Kaufman, Director
102 South Washington St., Carbondale, Illinois 62901

CSMP has as its goal the solution to the teacher training problems and the classroom organization problem by totally individualizing the mathematics curriculum, K-12. This program, if realized, offers considerably more than the IPI program because of its proposed objective of developing packages utilizing multi-media approaches, packages which will be created by CSMP itself.

Individual Mathematics Program, M. L. Clark, Director.
Australian Council for Educational Research, P. O. Box 210, Hawthorn, Victoria, Australia 3122, Miss Merle O'Donnell, Senior Advisory Officer.

IMP is designed as a modern program for Australian primary schools with an increased emphasis on the understanding of mathematical ideas as a basis for mathematical competence. It was prepared by class teachers under the general direction of the Australian Council for Educational Research, and was tried in pilot forms in actual classes. Recognizing that each child is different, the IMP provides material intended to allow each pupil to progress at a rate commensurate with his attainment.

Individualized Mathematics Instruction, Curt Oreberg, Director, IMI Project, Lejens vag 4, Braas, Sweden.

Initiated to meet the need for individualized instruction in mathematics in grades 7 and 8. Newsletters, reports, and descriptions of materials are available from the project director.

Individually Prescribed Instruction Project, Robert Scanlon, Director, Research for Better Schools, Philadelphia, Pennsylvania and Learning Research and Development Center, University of Pittsburgh, Pittsburgh, Pennsylvania.

IPI has several basic objectives. They are:

1. To enable each pupil to work at his own rate through units of study a learning sequence;
2. To develop in each pupil a demonstrable degree of mastery;
3. To develop self-initiation and self-direction of learning;
4. To foster the development of problem-solving through processes;
5. To encourage self-evaluation and motivation for learning.

The IPI system differs from most instruction systems because of:

1. Detailed specifications of educational objectives;
2. Organization of methods and materials to attain these objectives;
3. Careful determination of each pupil's present competence in a given
4. Provision for frequent monitoring of student performance, in order inform both pupil and teacher of progress toward an objective;
5. Individual daily evaluation and guidance of each pupil;
6. Continual evaluation and strengthening of the curriculum and instru procedures.

IPI at present is not on the commercial market, although many school sy are using the materials at a rather high price. The materials will be through commercial publishers soon, however, it is anticipated.

The Madison Project, Prof. Robert B. Davis, Director.
Webster College and Syracuse University.
Dr. Davis, Mathematics Dept., Smith Hall, Syracuse University, Syracuse

ally Prescribed Instruction Project, Robert Scanlon, Director,
for Better Schools, Philadelphia, Pennsylvania and Learning
and Development Center, University of Pittsburgh, Pittsburgh,
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commercial publishers soon, however, it is anticipated.

son Project, Prof. Robert B. Davis, Director.
College and Syracuse University.
s, Mathematics Dept., Smith Hall, Syracuse University, Syracuse, N. Y. 13210.

Originated in 1957 as a vehicle of exploring ways of revitalizing the teacher education program, the Madison Project is adapted both to enriching the classroom experience for the child and helping the teacher plan experiences that will involve the child both cognitively and affectively. The materials are designed to provide worthwhile laboratory situations and supplementary mathematics experiences. Dr. Davis explains what his project is all about when he says: "Guess--try--watch what happens--learn what to do next."

Mathematics Project of Sherbrooke, Z.P. Dienes, Director.
Project Mathematique de Sherbrooke, Centre do Recherches en Psycho-Mathematiques.
University of Serbrooke, 1382 rue Dominion, Sherbrooke, Quebec, Canada

The major objectives of the project are to consider:

1. The psychological problems of learning mathematics as from the standpoint of experimental psychology;
2. The methodological and the consequent pedagogical problems, as you meet them in the real classroom;
3. The curricular problems.

The curriculum emphasis is not traditional. Rather it emphasizes logic, geometric transformation, and non-decimal numeration systems. The emphasis is on mathematics rather than arithmetic.

Minnesota School Mathematics and Science Project, James H. Werntz, Director.
720 Washington Ave., S.E., Minneapolis, Minn. 55414.

The major objectives or purposes of MINNEMAST include the creation of a coordinated mathematics and science curriculum for elementary grades. Because of funding problems, the original intent of a curriculum for grades K-6 has been changed to K-3. MINNEMAST is the only national project whose main goal is to coordinate both mathematics and science into one curriculum.

Nuffield Mathematics Teaching Project, Geoffrey Matthews, Director.
12 Upper Belgrave St., London S.W.1., England.
(materials and titles now available from John Wiley and Sons, Inc., 605 Third Ave., New York, N. Y. 10016.)

The object of the Nuffield project is to produce a contemporary course for children from ages 5 to 13. It is being sponsored jointly by the Nuffield Foundation and the Schools Council, a widely representative educational body. The project started in September of 1964. All of the writing so far has been directed toward the teachers, in the form of Guides. There are no plans for producing texts for the pupils themselves. Emphasis is on learning by doing. The ultimate aim of the project is simple, according to Matthews, and can be stated: "to produce happy children (and happy teachers) capable of thinking for themselves." The film "I Do and I Understand", now in book form available from Wiley and Sons, is the single material most widely associated with the Nuffield project.

School Mathematics Study Group, E.G. Begle, Director
Cedar Hall, Stanford University, Stanford, California 94305.

Probably the most extensive mathematics project in the world to date, the major thrusts of the program include:

1. The development of programmed materials at the secondary level;
2. The production of films for elementary teachers;
3. The development of primary materials for the disadvantaged child;
4. The development of junior high school materials for the disadvantaged;
5. The National Longitudinal Study of Mathematics Achievement (NLSMA);
6. The development of content background books for elementary and secondary teachers;
7. The publication of a Newsletter;
8. The publication of an Abstracts journal;
9. A group of books designed for teachers and advanced secondary school students dealing with special topics;
10. A four-year longitudinal study to determine how children learn mathematics.

The major objective of SMSG since its inception at Yale University in 1958 has been a focus on the development of modern mathematics materials for grades K-12.

Stanford Computer-Assisted Instruction Projects, Patrick Suppes, Director, Institute for Mathematical Studies in the Social Sciences, Ventura Hall, Stanford University, Stanford, California 94305.

The project deals with various computer-assisted instructional programs for students, now through eighth grade, with ambitions of extending through 12th grade. One of the interesting aspects of the mathematics developmental work is the notion that a tutorial CAI program can be developed from a drill and practice CAI program. At present most of the Stanford CAI programs concern themselves with the drill and practice mode.

University of Illinois Arithmetic Project (at Education Development Center), David A. Page, Director.
Mrs. Patricia T. Kosinar, Assistant Director, University of Illinois Arithmetic Project, 372 Main St., Watertown, Massachusetts 02172.

The cental theme of the project is that the study of mathematics should be an adventure, requiring and deserving hard work. The project is not attempting to develop a systematic curriculum for any grade level, in the view that determining an adequate curriculum is not possible until more alternatives exist among which to choose. The emphasis is on things that the teacher can begin working with soon. The term "new mathematics" is avoided by the project. More properly, the project seeks novel ways of doing old mathematics--new structures or schemes within which can be found large numbers of interrelated problems revealing significant mathematical ideas.

Information on other developmental projects in elementary mathematics is available from:

Lockard, J. D., (editor), *Sixth Report of the International Clearinghouse on Science and Mathematics Curricular Developments*, College Park, Maryland: University of Maryland, Science Teaching Center, 1968.

Suydam, M. N., and Riedesel, C. A., *Interpretive Study of Research and Development in Elementary Mathematics*, Vol. III, *Developmental Projects*, University Park, Pennsylvania: The Pennsylvania State University, 1969. (Final report, Project No. 8-0586, USOE Bureau of Research.)

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Christensen, D.J., "The Effect of Discontinued Grade Reporting on Pupil Learning," *The Arithmetic Teacher*, 15 (December, 1968), 724-726.

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